



## The Reverse Logic of Resolving the Contradictions

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*Abstract:*

Contradictions have not only a negative role as a limiter of logical reasoning, they are a necessary element of the cognitive process at paradigmatic and philosophical levels. Reverse logic offers the mechanism of correct including of contradictory proposals in the structure of reasoning at these levels. At the base of the reverse logic lies the principle of demarcation between the procedure of obtaining the new propositions and the mechanism of transferring truth between propositions.

The principle of non-contradiction corresponds to traditional notions of reality: nothing can be *something* and can be *not something* at the same time (to be snow and to be not snow to be a quantum and to be not a quantum) or a single object can not simultaneously have the opposite qualities (to be high and low, positive and negative, salty and unsalted). In full compliance with the ontological obviousness of such an idea one of the main laws of logic is a law of non-contradiction. It is formulated both with respect to statements, "the statement and its negation can not both be true," or with respect to predicates "the opposite predicates can not be assigned to a single logical subject. "In most logical systems we derive the following principle "anything can follow from the contradiction" or the weaker one "the denial of any statement follows from the contradiction." Because of this, the systems, which violate the law of non-contradiction, and which may cause contradiction, should be treated as logically incorrect.

However, the development of mathematics and logic in the XX century has brought us to understanding that building the non-contradictory mathematics (which Hilbert was seeking) and generally non-contradicted and rich enough axiomatic theories is impossible (K.Godel). It became clear that despite the fact that the presence of a contradiction in logical systems should still be treated as a mistake, the possibility and necessity of such errors in general case should be taken as a regular inevitability. Therefore rose the task of rethinking a place of contradiction in logic: the detection of contradiction in the system must not be perceived as a death sentence for it, but only as an indication of the inevitable limitations of the system, the inadmissibility of the continuation of reasonings that led to a contradiction (L. Wittgenstein). The contradiction in the logical system is just a "stop" sign at the specific direction of its development, and it is not the lifelong deprivation of its logical rights. Consequently, the law of non-contradiction should be understood not as a prohibition of contradiction, but as the inadmissibility of any logical conclusions from them. To fulfill this requirement first of all the principles "anything can follow from the contradiction" and "the denial of any statement follows from the contradiction", which, in fact, realize the modern versions of paraconsistent logics, should be excluded from logical systems (N. da Costa, D. Battens, etc.). It should be noted that the interpretation of a contradiction as something unacceptable, and not as a source of arbitrariness is consistent with the concepts of reality: the contradiction is never realized ontologically. It can be said, that in the substantive reality the possibility of a contradiction in future determines the movement of the objects towards the circumvention of it.

But it should be noted that the value of a contradiction in the cognitive process is not limited to its role of a logical reasonings stopper. It means that the contradictions shouldn't be treated only as mistakes, paradoxes or inevitable disadvantage of logical systems. Indeed, apart from the fact that we fix the contradictions at the language level or within the frameworks of particular logical systems, that is, in the scope of the law of non-contradiction, we always face the contradictions at a higher paradigmatic level. To such paradigmatic contradictions can be attributed the contradictions between the statements of different logical systems (theories), between the scientific-theoretical and empirical statements. The example of such contradictions is the ratio of axioms in different geometries (eg, Euclidean and Lobachevskian), the comprehension of which led to introduction of the concept of the curvature of space, the paradox of the ultraviolet catastrophe, the resolution of which has given rise to quantum physics, a contradiction in the quantum-mechanical description of light as a wave and as a quantum, etc. The contradictions between the statements in different religious, world-outlook and political systems can be attributed to paradigmatic. It is clear that such contradictions can not be interpreted as logical errors or inevitable "defects" of complex systems. They, as well as the scientific and paradigmatic contradictions reflect some objective laws of describing the World.

It is obvious that the paradigmatic contradictions point not only to limitations of singular systems, but rather on the possible direction of their development, and even more on the necessity of creation the new systems, being the *meta-theories* with respect to the initial ones. The emergence of meta-theory – the geometry in spaces with non-zero curvature – removed the contradiction between the axioms of intersecting straight lines. It can be said that in contrast to linguistic and logical levels, where the contradictions are simply excluded, on the paradigmatic level there is a real resolution of the existing conflict: meta-theory confirms the validity and legality of the presence of the two initially contradictory statements in the sphere of knowledge.

The role of contradiction on the philosophical level, which is next after paradigmatic, is even more specific and interesting. There the contradictions are present not only as a boundary, a transition point, but also as an inherent element of logical systems. The striking examples of incorporating the contradictions into philosophical systems are the dialogues of Plato, Kant's *Critique of Pure Reason* with its antinomies, and, of course, Hegel's *Science of Logic*. The necessity of including the contradictions into philosophical systems becomes quite obvious if we present the paradigmatic level of knowledge as a substantive for the philosophical level. If we consider philosophy a sphere, which studies and describes the cognitive activity altogether, in such a sphere the paradigmatic contradictions, for example, the contradictions arising at the interface of scientific theories should be described as necessary elements that fix landmarks of the development of knowledge. The philosophical theory, pretending to describe theoretically the evolution of knowledge, should necessarily possess the mechanism for the inclusion of contradictions into its logical structure. Otherwise, we will have a lot of private descriptions of static projections of the cognitive process.

So, in contrast to the paradigmatic level at which the admissibility of contradictions is ensured by including their parties into different logical systems (in fact, there is a partition of the whole language area on closed non-contradictory fields of individual theories), on a philosophical level, where the contradictions are the subject of knowledge, they inevitably should be an element of the theory. That means, that the philosophical thinking, the logic of philosophical systems should not only permit (allow) the contradictions, but also imply the logical need for them. In fact, it is stated, that for an adequate cognitive thinking, inherently including the paradigmatic contradictions, the thinking itself should be contradictory.

However, with all understanding of place and role of a contradiction at the philosophical level of knowledge and even with the examples of theoretical systems with the contradictions, incorporated into them (Hegel) we still do not have any logically relevant mechanism for working

with them. The task is obvious: there is a need for the mechanism of assumption and resolving the contradiction, that is the formal procedure for establishing the truth of the initially contradictory proposals. Further I want to offer the option of solving the specified problem.

The idea of a possible mechanism for resolving a contradiction will be demonstrated with the help of the already mentioned example of ratio of the axioms of intersecting straight lines in Euclidean and Lobachevskian geometries.

We formulate the contradiction in the following way: "the space is of that kind (has such quality), that in a plane through a point outside a straight line we can draw one straight line, which does not intersect the given straight line" (S is P) and "the space is of that kind (has such quality), that in a plane through a point outside a straight line we can draw more than one straight line that does not intersect the given straight line" (S is not-P). We understand that each of the pair of statements is accepted as a true one in its logical system, but this affirmation of truth is not enough for us to resolve the contradiction at the paradigmatic level – at this level the statements are formally contradictory.

The history of cognition tells us the solution: for understanding the essence of the contradiction between the statements of different theories, we need to turn to meta-theory, combining the original geometries. Formally, it means that the meta-theory should have a statement from which the truth of both initially contradictory statements necessarily follows. It is reasonable to assume that the new statement should apply the logical subject of contradiction "the space" (S) and state its heterogeneity, duality, and remove the contradiction with the help of this. For example, a true statement in meta-theory can be formulated as follows: "there is a space with a different curvature: zero, positive, negative" (S<sub>0</sub>, S + S-). In substance, this statement affirms that the logical subject ("the space"), which is at the level of theories seems to us united, thus attributing the opposite predicates to it is interpreted as a contradiction, and at the level of meta-theory it is presented as a set of non-identical entities. Consequently, we must rewrite the original statements as follows: "the space with zero curvature is of that kind that in a plane through a point outside a straight line we can draw one straight line, which does not intersect the given straight line" (S<sub>0</sub> is P) and "the space with negative curvature is of that kind that in a plane through a point outside a straight line we can draw more than one straight line that does not intersect the given straight line" (S- is not-P). Thus, the original contradiction is removed.

So, we can conclude that, for formal resolving of a paradigmatic contradiction we should find such statement in the meta-theory, which would have a single logical subject with the original contradictory statements and affirmed the split and the plurality of the subject. Such scheme of argument can be called reverse-logical, since the transfer of the truth here is realized not from the initial statements (they are initially contradictory) to the following one, but vice versa, from the new statement to those, which were previously formulated. Considering, that the new statement has not been received as a result of a conclusion, it may be called *speculative*.

Let us try to analyze the functioning of the reverse-logical scheme at philosophical level. As an example, let us consider the initial contradiction in one of the most famous philosophical systems with the incorporated contradictions – in Hegel's *Science of Logic*.

First of all, it should be noted that Hegel rarely formulated contradictions in standard logical form, and we need to do it instead of him. It is clear that the statement "being is nothing" can not be interpreted as an assigning the predicate "nothing" to the logical subject of "being". Philosophy does not deal with objects outside the thinking at all: "being" is not a thing, not a subject, but a thought, and therefore another thought ("nothing") can not be assigned to it as a predicate. Therefore, in order to formulate a philosophical statement in the subject-predicate form, it is always more correct to introduce the thinking as a logical subject, and as a predicate – something, that is really belongs to thinking and can be assigned to it, that is a thought (a concept). The introduction of one concepts

as the logical subjects and assigning another concepts to them as the predicates, in my opinion, made the formulation and understanding of contradictions in philosophy very difficult.

Considering these simple thoughts, the statement about "being" as the first direct thought of the pure direct, yet not certain thinking should be formulated as follows: "the predicate 'being' is assigned to thinking as a logical subject." Furthermore, from this initial definition of thinking, because in it 'being' acts as a pure, immediate and uncertain, necessarily follows another statement "thinking is nothing." Or in an expanded form: "the predicate 'nothing' should be assigned to thinking, which has clear, immediate being as a predicate (that is, thinking the uncertain being).

So, we have a contradiction: thinking as a cogitative pure indefinite being at the same time is defined as a cogitative 'nothing'. Or in short: (1) "thinking is being" and at the same time (2) "thinking is nothing."

In this situation, in contrast to formal logic, in which a clear choice in favor of one of the conflicting statements should be done, we understand (feel) the truth of both statements. In addition, we have no reasons (rights) for such choice – we have no a priori set axioms (as in formal logical systems), with respect to which we could make a conclusion about the truth or falsity of the given statements. Moreover, the acceptance of one of the statements untrue automatically makes the second one untrue too. For example, if pure thinking is not "nothing"-thinking, it means that it is "something"-thinking, that means something specific, and therefore the predicate of "just being" cannot be assigned to it.

However, the logic must be logic and we can not stay in a situation of uncertainty. That means, that our challenge is to find a basis for resolving the contradiction, to find a new, speculative statement on the basis of which we could make a conclusion about the truth of originally contradictory statements. And such statement is "generated" by the original statements. Really, when we initially assigned the predicate "being" to thinking (presenting it as the thinking of pure direct being), we inevitably had to define it as "nothing" (as nothing-thinking), but, after this step (from being to nothing), we immediately got a new definition of thinking (which was not and could not be earlier): *the thinking as a transition*, as a pure movement, as *becoming*. That is, we can formulate a speculative statement as follows: "thinking has the predicate of becoming (the transition from being to nothing)." That means, thinking under the transition from "being"-thinking to "nothing"-thinking becomes not a direct (pure) thinking, but the certain thinking – the thinking which thinks. Although for the present it do not thinks about something outside itself, but only about itself as a pure possibility of thinking, shown as the movement, the becoming.

We also can discourse as follows: if a statement of pure being is a kind of direct *beginning* of thinking and its initial definition, the second statement of nothing-thinking can be imagined as *the end* of thinking (the end of this immediate thinking). But since in this view of the beginning and the end the thinking itself is revealed, is defined as the unfolding, it acts not as pure and uncertain, but as having acquired the certainty, as the transition from the beginning to the end, as the becoming.

So, we have three statements: two initial and contradictory – "thinking is being" and "thinking is nothing" – and one speculative "thinking is becoming". Here it should be noted that the thinking, which was a logical subject in the first two statements is not identical to the subject-thinking in the speculative statement, in which it has acquired the duration in time and became distinguished in itself. Now being-thinking, and nothing-thinking are acting as points (starting and end) of the becoming-thinking. With respect to speculative statement we can reformulate the original contradictory statements as follows: "thinking as a direct one is a thinking of pure being" and "thinking, which is mediated with the transition, the becoming, is thinking of nothing". That means, that the statement of thinking as becoming removes and permits (allows), the initial contradiction, affirming the non-identity of the subjects of the original statements. Thus, we have implemented and confirmed the reverse-logical scheme: *getting a speculative statement from a pair*

*of contradictory statements can be considered valid only if the speculative statement removes the divergence between the initial statements, presenting their logical subjects as distinguished ones, and thus the truth of both initial statements follows from the truth of a speculative statement.*

Let us consider some important points of the proposed logic. In the scheme of reverse logic we do not have any a priori true statement: the truth of initial statements follows only from the truth of a speculative one and the truth of the latter "hangs in mid-air". It can simply be accepted, as in classical logical systems is accepted the truth of axioms. And if we continue the discussion in the same way – revealing a contradiction, getting a new speculative statement, resolving the contradiction with the help of it – then the first speculative statement gains a legal status of a true one in the chain of reasonings.

The truth of the statements in this chain of reasonings will always depend on the truth of the last speculative statement. But there is nothing unnatural in it: the situation is similar to the situation in classical logics, in which the truth of all statements is directly inherited from only the admitted truth of axioms. That means, that in both logics the truth of the chain of reasonings in any case depends on an axiomatic statement: in classical systems this statement is at the beginning of reasonings, and in a system built according to reverse logic – in the end. But, there is also a significant difference: the last in the chain (reverse-axiomatic) statement is not the result of our arbitrary choice, but a consequence, the result of the development of the logical system itself. And, if under the expanding of this chain the initial immediate statement can be "received", we can loop the system and, in substance, remove its uncertainty, attachment to the conclusion about the truth of one statement (which Hegel was seeking while building his Logic).

It should be noted that the scheme of reverse logic is quite rational, that means, it does not allow any arbitrariness in the transition to a speculative statement, because it has strict requirements: to declare the distinguishing of a logical subject and an obligation of pursuing the truth of original statements from its truth.

However, there is an uncertain, undetermined, creative point in the very search of a speculative statement – because it does not follow logically from some true statements (our initial statements are contradictory). But it is clear that this point of free creativity is incorporated to the systems built according to classical logic – we always have it on the stage of choosing the axioms. And the necessity of a permanent choice in the reverse-logical scheme affirms its creative specificity.

The next point concerns the role and place of a contradiction in logical systems. It should be noted that in classical logics the prohibition of a contradiction is associated with the requirement of the unambiguity of truth transmission. In fact, the law of contradiction states the impossibility of logical transition from untrue statements to true ones. And since one of the contradictory statements is necessarily false, and the transfer of truth is possible only in forward direction, the contradiction is certainly prohibited. And what do we have in reverse-logical scheme? In it the transfer of truth from statement to statement occurs only from a speculative statement to the initial ones. Consequently, the presence of a contradiction at a certain stage of logical reasoning does not imply the possibility of the transition from the untrue statements to true ones. Not to mention the fact that the very contradiction is resolved by further reasonings.

In connection with the above matter, there is a need to expand the understanding of logic, dividing in it the procedures of getting the new statements and the transferring of the truth between the statements. In classical logic, these procedures are combined: the withdrawal of a new statement automatically implies the transferring to it the truth, which is initially stored in the system of axioms. The proposed in the reverse logic variant of separation of the truth transferring mechanism and the mechanism of formulating the new statements allows us to "work" with the contradictions within the frameworks of one logical system: it should be noted that although the contradictions are removed (resolved) with the help of speculative statements, they can not be taken out of the system,

as they constitute its essential element – the speculative statement is formally attached to them. But the very presence of a contradiction does not affect the truth of the system, because there is no transfer of the truth from contradictory statements to any other statements.

Let us demonstrate the functioning of reverse logic with the help of another, more simple than Hegelian, example. Let us consider the two contradictory statements: "philosophical thinking is scientific" and "philosophical thinking is not scientific" ( $S$  is  $P$  and  $S$  is not- $P$ ). We are looking for a speculative statement, which fulfills the requirements of reverse logic: it must have the same logical subject with the contradictory statements, affirm its heterogeneity and the truth of the initial statements should go from the assumption of its truth. A possible version: "philosophical thinking has the thinking of the philosopher as its subject" or "philosophical thinking is thinking of thinking." That means, that in the speculative statement the initial logical subject is divided into two subjects, "the thinking as a method" ( $S_m$ ) – the thing with the help of which the philosopher thinks and "the thinking as a subject" ( $S_s$ ) – the thing of which philosopher thinks. Further we have two implications: (1) "if the philosophical thinking is the knowledge by thinking (rational), it is scientific (in contrast to artistic, religious, etc.)," or "philosophical thinking as a method is a scientific thinking" ( $S_m$  is  $P$ ); and (2) "if the subject of philosophical thinking is the thinking of the philosopher itself, that means the clearly unreproducible, unique object, in this case the philosophical thinking is not scientific" or "philosophical thinking as a subject is not scientific" ( $S_s$  is not- $P$ ). So, we have a system of three true statements built on a pair of initially contradictory statements.

In conclusion, I would like to note the similarity of reverse-logical scheme with the *abduction* – a procedure of searching for the true hypotheses, proposed by C.S. Peirce. Both the abduction and the reverse logic are designed to formalize the creative thinking, as a result of which the credible hypothesis, explaining new facts should appear, or the speculative statement, affirming the truth of previous contradictory statements. In both cases the received statement is not a logical consequence of initial data. In both cases there is a coordination of the assumed truth of a statement with the existing statements. However, there is a significant difference between the abduction and the reverse logic. The hypothesis, resulting from abduction, although appeared *after* fixing the initial data, eventually logically takes the place of a message. That means that separation of procedures of obtaining the statement and transferring its truth to other statements is realized only outside the logical system – the final system (theory) is formed by classical logical rules. In the system, which is built according to reverse logic, the transfer of truth from the late (speculative) statement to previous initially contradictory statements, saving their order in the reasonings is formally legalized.

It should also be noted that the very fact of the "reverseness", determination from the future, realized in the scheme of reverse logic corresponds with our understanding of the specificity of the cognition process: while trying to understand creatively, we intuitively compare our thoughts with the idea, that is not yet "caught", not formulated, but we definitely know, that it exists and are sure of its truth.

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