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What is Time in Some Modern Physics Theories: Interpretation Problems

Ivan A. Karpenko

National Research University Higher School of Economics,
School of Philosophy,
Laboratory for Philosophical Studies,
Moscow, Russia

e-mail: gobzev@yandex.ru

Abstract:
The article deals with the problem of time in the context of several theories of modern physics. This fundamental concept inevitably arises in physical theories, but so far there is no adequate description of it in the philosophy of science. In the theory of relativity, quantum field theory, Standard Model of particle physics, theory of loop quantum gravity, superstring theory and other most recent theories the idea of time is shown explicitly or not. Sometimes, such as in the special theory of relativity, it plays a significant role and sometimes it does not. But anyway it exists and is implied by the content of the theory, which in some cases directly includes its mathematical tools. Fundamental difference of space-time processes in microcosm and macrocosm is of particular importance for solving the problem. In this regard, a need to understand the time in the way it appears in modern physics, to describe it in the language of philosophy arises (satisfactory for time description mathematical tools also do not exist). This will give an opportunity to get closer to the answer on question of time characteristics. And even if we do not obtain the exact answer, we will still be able to formulate the right question about its nature. For this purpose, the present research carries out analysis of the key theories of modern physics with regard to historical and scientific, historical and philosophical perspectives. In some cases, this gives an opportunity to detect the succession of the associated with time perception ideas, their development, as well as the origination of fundamentally new ones. During the analysis, the correct characteristics of time are formulated from the point of view of physical theory and the attempt to state the nature of time is made. On the ground of conducted research, the conclusions about current state of the problem and its future solution perspectives are drawn.
Keywords: time, philosophy of science, modern physics, concept of space and time.

1. Introduction

The problem of time is not an entirely physical problem. Physics itself does not contain a "time theory". In the sense that there have not been made any direct attempts to find the natural-science answer to what is the time. Nevertheless, the concept of time emerges in science anyway and still requires an explanation. Time fulfills an important role in physics of XX and XXI centuries, though often a hidden role. This applies to both theories of macrocosm and theories of microcosm. In the theory of relativity time is established as a secondary feature, a derivative of the velocity and mass, but nevertheless, it exists (although as an illusion) and evokes the need of its philosophical interpretation. In quantum field theory time also (though implicitly) occurs in relation to the interpretation of the experiment results – for example, "where the particle was *before* observation of it". This "before" exactly indicates the presence of time (more precisely, the observer`s perception of its presence).

In further theories, which have been the attempts to solve the problem of incompatibility of general relativity theory and quantum mechanics, such as the theory of loop quantum gravity, superstring theory, Shape Dynamics and others, time also fulfills a definite role and evokes the question of its explanation in the frameworks of these theories again. I.e. what meaning of this "time" term has been chosen to be used?

This article deals with the problem of time in the context of several theories of modern physics. In particular, it attempts to answer the question of what the time is in relation to the philosophy of physics (physics itself does not have such an answer). During the research the historical and philosophical practice of terms interpretation has been reviewed.

2. Problem Statement

As noted by Gerald James Whitrow, the author of ones of the fundamental works on time matter [45], [46], [47], time geometrization is basically typical for physics. Albert Einstein has noted the same [12, p. 141]. If we have got used to conceive space with coordinate system, in terms of dimensions and distances[1], then there are no specific temporal concepts, which would characterize time itself. To some extent, this explains the difficulty in interpretation of various physics theories usage of time concept – it is hard to interpret something that has no endemic characteristics. Because of this exact absence of its essential attributes time has become the subject for geometrization. This becomes particularly obvious in the theory of relativity and in the description of time in Minkowski space (constructed on the basis of Henri Poincaré [39, pp. 129-176] and Einstein [11, pp. 891-921] works by Hermann Minkowski [34,pp. 75-88]), where time constitutes the fourth coordinate and products light velocity and event time. In general, in the theory of relativity the problem is solved radically – time has been almost eliminated, it is nothing more than an illusion of perception. However, in this case the situation is the same not easy and time rejection causes serious difficulties with universe evolution (these difficulties are to be discussed later).

The traditional time discourse creates purely spatial questions: if time is discrete or continuous, whether it can be identified with change (sequence of events) and duration, whether it is objective or subjective, relative or absolute, fundamental or not, etc. These issues result from the classical intellectual intuition, revealed back at works of Zeno, then Aristotle, the Stoics and later Augustine. That intuition has been also noteworthy preserved almost unaltered at Galileo Galilei and Isaac Newton works.

It is significant that in these cases, where peculiarity of time is stated (ones of the latest works, as an example, Lee Smolin books [42],[32], which proves the fundamental nature of time and its independence of spatial terms), time definition still remains unclear.

As a consequence, the most important question on time matter seems to be the following: does time even exist? The present research certainly does not contemplate on giving an ultimate answer. However, the analysis of the time concept and its possible characteristics, emerging in modern physical theories, allows close approaching to the answer.

3. Brief Historical Background of the Problem

Concerning discreteness and continuity the problem of time was, apparently, first described by Zeno of Elea[2] in his famous paradoxes. The *Achilles*, *Dichotomy, Arrow* and *Stadium* paradoxes are aimed against motion. The first two of them deny motion, if space and time are viewed as continuous, the latter two, when both space and time are discrete. Alexander Koyre in *Notes on the Zeno's paradoxes* [27] has made an important observation that these paradoxes, in fact, are not related to motion and only concern it insofar as the motion takes place in time and space. Then an important question can be formulated: is movement (or any change) possible out of time? We shall return to this matter in the following analysis.

Further, Koyre reveals that all the four arguments allow a double interpretation, which means that the *Achilles* and *Dichotomy* paradoxes stay valid if we consider space and time as discrete. Similarly, if we consider the *Arrow* and *Stadium* from the point of space and time continuity, the paradoxes still remain unsolvable.

The next step suggests that the concepts of space and time can be ignored and instead the mathematical continuum should be considered, without depriving the paradoxes of their original meaning. After applying such transformations Koyre concluded that the problem is much more intense than the experts and critics of Zeno had imagined and discovered (after paradoxes translation into the language of mathematics) that these paradoxes are rooted in a hidden form in any geometric theorem, in any algebraic formula, in any arithmetic assertion. I.e. the problem is inherent in mathematics and geometry, but within this mathematical approach it ceases to be a problem, since in terms of mathematics there is nothing paradoxical in continuity. It may lead to a conclusion that our traditional intuitive ideas about motion, time and space require serious consideration, only then the paradoxes will cease to be paradoxical. Or (which is quite the same) the paradoxes were generated by misconceptions about movement, space and time.

Another issue, which refers to Zeno's paradoxes, is whether time should actually "flow". Bertrand Russell [41,p. 813] has not seen any contradiction in the arrow seemingly leaping from one position to another. This means that motion in time can be understood as a change of position, following the example of the trotting second hand. Quick change of position is perceived as smooth motion. Viewed this way, motion lacks what we intuitively understand as motion. At any given moment the arrow resides in a new place. However, we can consider this motion.

In *Timaeus* Plato describes time as a rotating similarity of eternity [37, 37a-38c], apparently, in accordance with the cyclic tradition. Real time is frozen eternity; its similarity is the time of rotating sky, a move from number to number.

The first attempt to scientifically analyze the concept of time might have been made by Aristotle in *Physics*. He was naturally dissatisfied by Plato's approach, primarily because he could not accept Plato's Ideas as prototypes of the observed reality. According to Aristotle, time and motion are related, but not identical. In his words, "time is the number of motion" [1, IV, 11] and it is continuous. Motion is measured by time, and time is determined by motion. But a necessary condition of motion is space, which means that there is no time without space. Perfect motion is circular (here we again witness the tradition of cyclical time concept, the movement of celestial bodies, the circle of life, the change of seasons, etc.).

Plotinus disagrees with Aristotle. According to him, time (following Plato) is derived from eternity. But he does not perceive time as motion or measure of motion (not the number of motion) [38, III, 7].

His objections towards time as motion or number of motion can be briefly explained as follows: motion presupposes time, but time does not require motion and can fully coincide with quiescence. Which means that "if we assume the possibility of motion out of time, then equating time to it makes it even more incomprehensible, as consequently, time would be one thing and motion quite another" [30,p. 442]. So, time is one thing, and motion is another. Also Plotinus shows that since motions can be different, i.e. different distances can be covered in the same period of time, then times must be different too, which is impossible (it is interesting that idea of time relativity appears here by now, derived from motion, but though denied). Consequently, distance cannot be conceived as time. This can be interpreted as a protest against the spatial nature of time.

Later Plotinus claims that time is not a number, arguing that anything can be measured with numbers, not just time, so time is time, and a number is a number. In terms of a definition, according to Plotinus, time is the length of an eternal life of a soul (but again length is a spatial characteristic).

A special place in the time studies is occupied by famous reflections by Augustine. He claims that time is not motion, because there is no real past and no real future, but there is present. However, this present lacks durability; it's momentary, so, in fact, it does not exist. But all the three times – the past, the present and the future – exist in human soul. We wish to interpret this as the statement of illusiveness, subjectivity of time, but Augustine does not explicitly say this. Time still exists and he expresses it in his famous formula, "time exists only because it tends to disappear" [2, 111.XIV.17]. He rather reveals the psychology of time perception – occurring in one's soul an image of the present allows thinking about the past and the future following the model of this present (a kind of induction). Augustine has also introduced a novelty, which can be directly attributed to physics. Discussing a popular issue of that time, "What had God been doing before he created the world?", he boldly declares: nothing. For the simple reason that time has been created together with the world, how can we speak of before and after, if there was no time? Without time these concepts simply do not make sense. Thus Augustine states the following idea: there is no external fundamental eternal time as an arena for physical laws. Time occurs together with the Universe.

In his commentary to Plato's *Timaeus* Neo-Platonist Proclus (apparently, following Iamblichus's ideas) [40] has developed kind of time and eternity dialectics. Time for him is duration, fluidity, continuity. Time is (again in Plato`s spirit) a motile image of eternity and eternity is a fixed image of time. Time is associated with motion and for its flow something to force every event into motion is required, as each event needs something to cause its movement. The initial cause of motion is eternity.

Damascius has developed these ideas working on the problem of the essence of time [30, pp. 436-439]. But he has introduced the time quantized. If time consists of non-durable moments of the present, it would be impossible to make them up into a proper duration. The same would be adding even an infinite number of non-dimensional pieces – you still get nothing. Thus, the time should consist of indivisible segments of the present, which all have duration. In other words, time leaps. Damascius explains this by giving an example of human thinking: a thought seems to be continuous; however, it cannot contemplate all at once: first, it is aimed at one thing, then at another, and so on. This attitude can be interpreted as an attempt to prove the discreteness of time. Moreover, the velocity of the time stick-slip motions results in the fact that different movements have different time (which was denied by Plotinus). In fact, this means that time is relative and its relativity is determined by the motion velocity (though, obviously, Damascius was guided by completely different grounds for his intuition, compared to the later relativists).

Philosophers and scientists of later times (the Middle Ages and Modern Period) have deepened the already existing ideas or argued with them, but in general, have remained within the same agenda. However, some other views on time that have defined the modern discourse are worth mentioning, meaning the concepts of Kant, Leibniz, Newton and Einstein.

Newton has insisted on the objective status of time. However, he avoids metaphysics, he does not define the time. But he claims it to be absolute. "Absolute, true and mathematical time, of itself and from its own nature flows equably without regard to anything external and by another name is called duration" [35, p. 30]. Time exists and it is duration. While with the aim of proving the absolute space Newton has conducted an experiment with a rotating bucket (which, however, proves nothing, as noted by Ernst Mach [31, pp. 198-199]), but to prove the existence of absolute time he has had no reasoned arguments. Gottfried Leibniz, the opponent of Newton, on the contrary, has postulated the relativity of time, deriving it from the principle of sufficient reason and the principle of identity of indiscernible[3]. One can say that he has deepened Augustine's argumentation on the matter of what God had been doing before the he created the world [29, p. 56]: without events and things (of the world) there is no time. But this does not mean that Leibniz has denied time; it does exist and, moreover, it is universal (it makes it obvious that there is nothing in common between Leibniz's and Einstein's concepts of time).

Kant has put the objectivity of time under the question once again. It is interesting how Losev makes a rather sharp remark about Kant's idea on apriority of time, arguing that it was entirely borrowed from Plotinus [30, p. 447]. According to Kant, time is an a priori form of sensuality, which enables us to organize the experience of interaction with the world in our perception [21, pp. 56-58]. It is nonobjective, there is no time itself. Therefore, discussions on the nature of time, its essence or properties are meaningless – they must be limited to our perception, to the activity of consciousness[4].

Developing the relativity concept, Einstein has formulated the last fundamentally new idea about time, which corresponds to the classical intuition (intuition about macrocosm only, to quantum mechanics, for instance, this intuition cannot be applied). The fundamental novelty of his step is the assertion of relativity of simultaneity, where under the same events seem to be differently separated in time for different observers, depending on the movement velocity (including direction) of the last.

4. The Current State of the Problem

Each of the mentioned time theories gives rise to specific complexities, which besides originate not from the viewpoint of physics, where utilized mathematical tools on the assumption of consistency ensure adequate descriptions. Difficulties arise while attempting to perform philosophical analysis.

In modern physics the notion of time discreteness is rather popular. This does not anyhow contradict the fact of mathematical time continuity: a mathematical theory that would virtually explain time does not exist. Therefore, following the ideas of Russell and Zeno, one can assume that a time quantum, "chronon", is a Planckian quantity. Upon this assumption, however, two issues arise. The first and the most obvious issue is the debatable representation of a time unit as a quantum, i.e. something that has fixed dimensions. In this case time appears to be just a particular spatial dimension, where the specific movement takes place. Thereat one can say that between point (event) A and point (event) B there is a certain number of time quanta. In theory, there is no paradox here – movement in time can be represented as a saltatory (quantized) motion in quiescent state at the maximum permissible velocity. That is, when something is stationary in a point of three-dimensional space it can be interpreted within the STR in such a way that it moves at the velocity of light in time and, therefore, moves in the space, because the space changes over time run.

This raises another problem. If time is quantized, what do the changes of space in time mean? Does the time exist in the "gaps" between time quanta? If it does, we would have to admit the existence of space out of the time intervals, which seems absurd, because it would make us admit another kind of time and so on, and so forth, which leads us to a vicious circle. Instead, if time does not exist in the "gaps", the world is created all over again at every moment, which also contradicts our intellectual intuition[5].

As an alternative to divisible and infinitely divisible time may serve absolutely indivisible time. It is not quite clear what it means and how we could describe the past and the future under this concept. Probably, this is a representation of time as a single moment (meaning that everything exists simultaneously, but in this case it's unclear why we would even need the category of time after all). The concept with more apprehensible continuity at least presupposes durability.

Durability, as a characteristic of time, is in close relation to the notion of locality. In classical physics, starting with the works by Galileo, Newton and up to Einstein, time is described as local. This basically corresponds to our intuition. In general, the concept of locality concerns space, of course. Its main idea is that in order to get from point A to point B it is necessary to cover a certain distance. However, since the maximum possible velocity is finite (the velocity of light), locality implies the need to spend certain time to cover any distance. It is interesting that Newton's universe is not entirely local, contrary to common belief – in his theory gravity extends instantaneously. In the relativity theory gravity has a fixed velocity – the velocity of light.

In quantum mechanics the non-locality appears (surprisingly, it was established again by Einstein [13, pp. 777-780], who believed that this way he would demonstrate the insufficiency of the quantum theory). Non-locality means that in order to get from point A to point B photons do not have to cover any distance, they immediately reappear at point B, therefore it takes no time. A broader understanding of the non-locality [33], [9] presumes that there are non-local connections between elementary particles in the Universe and the more connections are there, the more strongly marked are the other dimensions. In other words, if in order to relocate macroobjects in addition to the three dimensions one or many more dimensions were discovered, it would mean the presence of non-locality. It is a curious crossover between quantum mechanics and the relativity theory. Within the relativity theory, as previously noted, there is no time itself, but there is space-time. Thus, every motion is a motion in space-time. If we assume the existence of extra dimensions (which, incidentally, Einstein has already tried on the basis of the Kaluza-Klein theory [20, pp. 966-972], [26, pp. 895-906]) and add the quantum-mechanical non-locality, it turns out that non-locality means movement in other dimensions[6]. Experiments to prove non-locality (quantum teleportation) have already been carried out repeatedly with the quite recent last one [8, pp. 775-778]. However, interpretation of their results is a big challenge. On the basis of them, it is possible to conclude that the concepts of distance (as a characteristic of space) and continuity (as a characteristic of time) are invalid, as they are only a matter of our perception. Furthermore, there is only one place where everything happens (if only we can talk about a place here). However, quantum teleportation involves the creation of a duplicate of the original object *somewhere else*. The presence of this other place immediately undermines the idea of illusory distance. Otherwise, we should assume that this is not a different place, but the same one; and the photon is exactly the same; there are not two of them, but one. But then the question arises: did the teleportation happen? What kind of manipulations have the experimenters performed if nothing has changed compared to the original state? The idea that the object is the same one is consistent with the Leibniz's identity of indiscernible principle.

The problem becomes irrelevant within Newton's absolute space (and Einstein's space[7]). It's not necessary to consider that non-locality means being in one and the same place, it is all about photons which instantly share information across any distance. Yes, the photons are indiscernible, but Leibniz's principle remains inviolate because the various positions in space are also a characteristic of a photon (although, in this case, the function of space is uncertain, it becomes even more artificial than the famous ether of the old physics, which at least was required for the propagation of light waves with a finite velocity). But time is eliminated completely. Thus, combining the relative and quantum interpretations of non-locality shows that time does not exist and it is pointless to talk about any of its characteristics, features or properties. For more information on non-locality and the measurement problem, see [24], [22, pp. 16-28].

It is possible to view time as a sequence of events. Only in modern physics processes at the microscopic level are called events. For example, as a first event you can take the first fluctuation or decay of a particle which has launched a chain reaction of events, some of which are cause-related, which is called time (in this case the beginning of time is not necessarily connected with the unique creation). This raises objections (also not new) that time is still not a sequence of events, because the events are still occurring in time, not the time in events (or time is not motion, but motion takes place in time).

We shall enlarge upon the issue of the "flow" of time. The relativity theory Universe is often called "block" universe, meaning a single space-time in which there is, indeed, no time. The block can be "cut" under different angles (the velocity and direction of motion corresponds to "cutting") – this defines the relativity of simultaneity. For different observers different events will be perceived as happening in a different temporal order; what happened earlier for some observers, happened later for others. The order of events is unchanged only within the boundaries of a single light cone. Events beyond the one cone may not be causally related. Here the key word is "to be perceived", as within a block Universe all events are set up, they occur, and our perception depending on certain conditions allows us to notice them in a certain order. This model excludes the flow of time, the sequence of events cannot be called time either, therefore it lacks continuity as well.

Such circumstances are not accepted by as completely satisfactory. The reason is the presence of so-called "arrow of time". Its specificity is that no matter where and how the observers move, they will observe its one and only direction – from the past to the future (in the sense that the past is different from the future). Although, the laws of physics are reversible in time, reality is irreversible. Order turns into chaos and the opposite is very seldom (though this must occur with the same frequency). This is the well-known second law of thermodynamics and its implications, first described by Rudolf Clausius [10, pp. 481-506], and studied by Ludwig Boltzmann [7]. Entropy always increases. Even if self-organization, the growth of the order take place, it presumes the use of energy and the spent sufficient energy leads to the release of insufficient energy (heat), and entropy (as a measure of disorder) is always greater than the increase of the order.

In this case it is important that the second law of thermodynamics might be considered as an evidence of the time flow or, more boldly, as a description of time itself. In this context time could be comprehended as a transition from less probable states to more probable – and the most probable state is the state of equilibrium. However, in this case we would have to admit that at the moment of equilibrium time stops. This is obviously not true, because in any state of equilibrium fluctuations occur, reducing the entropy in the area around them and then again it leads to a decrease in the order degree. Thus, the second law is not time, but it works in time (in the words of Aristotle, motion in time). It points at the arrow of time.

The arrow of time presupposes asymmetry of the universe in time: if the past is fundamentally different from the future, there must have been some special initial conditions. The choice of initial conditions (in the inflation model, for example) is random to a great extent. The point is that knowing the current state of the universe, it is impossible to reconstruct its original state; it could have reached its present state in many different ways. An important role in the history is played by random events (nondeterminate somehow) – fluctuations. Therefore, the choice of initial conditions is large enough. And even if we ever discover what they were exactly, the question of why they were such and not other, since those other might have been, will remain. In fact, the "accidentally" answer is always possible (which does not withdrawn the question "why is an accident possible?")

Another problem is that the recognition of the arrow of time implies the choice of initial conditions with a high degree of order. If the most probable state is equilibrium, then for something that we are observing at the moment the initial state must have been non-equilibrium. Or it was an equilibrium, but from time to time large-scale fluctuations occur in various parts of the Universe, which increase the organization level (the idea of Boltzmann) [28]. This assumption requires eternal past, since the probability of giant fluctuations is extremely low and they could have hardly occurred in 14 billion years. If we choose initial state after the Big Bang, it is necessary to explain, where the original order had arisen, which set the direction of the arrow of time. However, gravity is the very factor of the order in the initial conditions. The initial state of equilibrium after the Big Bang cannot be same equilibrium in the presence of gravitational interaction, which makes the elements pull up together to form complex structures. In this case, one has to think that time is somehow connected with gravity and, perhaps, is derived from it (in a certain way the GR confirms this). If so, we should clarify what exactly the gravity is. If it is treated in the sense of the GR, as the space-time curvature, time is excluded again. It is possible to consider gravity as a result of real carrier particles (gravitons) actions, as the superstring theory, for example, has it and predicts their existence. In this case, time will have to be connected not only to the force of gravity, but, apparently, to repulsive gravity, the cosmological constant, too.

Another modern concept which eliminates time views the Universe as a hologram. This approach has emerged from the black hole studies begun by Jacob Bekenstein [4] and Stephen Hawking [18], was continued by Gerard`t Hooft [43, p. 621] and Leornard Susskind [44] and was completed by Edward Witten [48] and Juan Maldacena [19]. T`Hooft and Saaskind showed that all the information about any object can be recorded on its surface area, i.e. the information within the area is always smaller than the surface. This suggests that the arena of physical laws is just the border and the observed three-dimensional reality is a holographic projection[8]. Maldacena, a string theorist, following the principles of Witten, has revealed the possibility of a dual description of reality. His string theory (the strings in the beam) is identical to the quantum field theory. This has become possible because the same mathematical vocabulary is used to describe what is happening inside the Maldacena's world[9] and on the border of this world (the actual quantum field theory). Thus, both theories are essentially the same, but they describe reality from different perspectives. The essence of the concept is that you can describe what is happening *inside* by what is happening outside, on the border area. For example, it may mean that a black hole is a holographic projection of gas on its surface. Then black holes may appear to be quite trivial objects[10]. Ultimately, the universe can be described as a hologram, i.e., as a projection from a distant flat surface.

What is the role of time in such a model of describing reality? Should we consider that time is also a projection (and if we consider time to be a derivative from the laws of physics and arising from them, rather than preceding them, a projection of what is time then)? In the spirit of Plato and Platonists one can say that it is "the projection of eternity", but from the viewpoint of physics, the answer, of course, is not concrete enough. Probably, it would be right to say that there is no time on the surface, time is just a property of three-dimensional projection. The projection is moving, and here, as Heraclitus put it, "everything flows, everything changes", but the boundary surface remains unchanged (because it is atemporal). This means that time is not fundamental and is derived from something else that is encoded on the remote surface. And currently it is not clear what could that something be.

Another option, which is in a better compliance with the string theory, is that time (and space) is a predetermined pattern, a stage for events. One version of a cyclic universe (or multiverse), which has been proposed in the superstring theory, considers time precisely this way. In the model, proposed by Paul Steinhardt and his colleagues [25], our universe has been discussed as a three-dimensional brane, located in the space of a higher dimension. From time to time, a collision with other brane-universes[11]may occur, which means end of these universes and arising of new ones.

Another cyclic theory within more classical beliefs has been proposed by Penrose [36]. His concept suggests that a new Universe is the result of fluctuation (in fact another Big Bang), which is inevitable *an infinite* time later after reaching the thermal equilibrium. When this latter is accomplished, later universe becomes indistinguishable from the earlier one. Thus, the end becomes a new beginning. Interestingly, this theory, despite of the obviously strange need in expiration of infinite time, is able to be verified. The detection of gravitational waves and concentric circles from the collision of several black hole pairs may speak in its favor. There is evidence that such data have been obtained [14].

But any cyclic model, no matter what it is based on, requires the presence of predetermined time outside of universe, which is not going to arise and die along with the universe. Indeed, the cyclic scenario makes no sense when time appears with the emergence of the universe. How can one claim the previous existence of universes, if their time had disappeared together with them? If there are timeless intervals between universes (which fact is absurd of itself), we cannot use terms "before", "was", etc.

Such scenarios only complicate the problem of time. Time here appears to be a certain fundamental value, which is wittingly impossible to perceive, as it is placed outside the world. The same applies to the different concepts of multiverse [23]. If we exclude the external time and leave only the proper time of each world, the question arises: how do the worlds relate to each other in time? The theory of eternal inflation [17] raises another question: is there time in the inflaton field or Plato`s eternity reigns over it and time appears only together with the worlds, in "bubble" universes? In this case, timeless would separate worlds, which cannot be comparable over time and we can neither say that the worlds have different time, nor that they have the same one. This issue also does not lose its relevance in the case of Everett's many-worlds interpretation, string landscape and some other multiverse theories.

The interpretation of time in quantum field theory is also quite special. The Schrödinger equation describes the wave function of the particle before it is measured, at the very moment of measurement the wave function collapses and the macrocosm now dictates its rules to situation. Thus, time plays a key role in the act of measurement. Measurement changes the future. Before the measurement the past of a particle is blurry (it may be anywhere with a number of the most likely positions and, more precisely, it may by anywhere at the same time if hidden variables are not allowed). At the moment of measurement the particle is detected somewhere and the rules of a microcosm no longer work for it. If the measurement had not been carried out, the particle would have been further described by a wave function. In this situation the moment of measurement has special, determining the future powers – this is the very moment of present, which separates the future from the past. On its only basis we cannot reconstruct the past and are able to only statistically predict the future. The past and the future are always blurred, only the present exists.

From the viewpoint of the hidden variable theories [6] the act of measurement itself is nothing special. It just allows detecting the previously unknown location of a particle. The concept of Hugh Everett [15] suggests that measurement also does not have any special status and appears to be one of the possible realizations in parallel universes. Most of the other interpretations of quantum mechanics also avoid the problem of measurement [5].

The standard quantum field theory, however, has to recognize a selected moment in time (it is important that it is chosen by the observer, i.e. a macrocosm object). In a microcosm time reveals itself only in interaction with the macrocosm, when the observer comes to intervene. So, does this mean that the concept of time is only meaningful in macrocosm and it makes no sense in microcosm? This point of view has existed for quite a long time. In fact, the classical representations of space and time at the scale of Planckian quantities are probably pointless. However, for lack of better options physicists and mathematicians have to use traditional systems of coordinates and clock. To be more precise, the classic time and space makes sense, but only at the moment of transition from micro- to macrolevel.

As an alternative to the general relativity theory a number of physicists [16], [3] suggest Shape Dynamics. The fundamental difference of the Shape Dynamics theory is that time here is considered to be universal, while space is relative. This means that there is a distinguished observer and, accordingly, allotted time. The relativity of space means that in different parts of the Universe the size of similar objects may be different or more precisely, the concept of size over long distances has no independent meaning, just as the concept of simultaneity of events in the theory of relativity. Global time and simultaneous observation are possible, because, for example, a universal frame of reference had been chosen – the microwave background radiation. The observers will register its one and the same temperature in all directions of the universe, so there are separated observers (which fact, however, brings asymmetry into GRT). It is important that Shape Dynamics is a dual description of GRT; the relativity of time is replaced by the relativity of space, two theories are equivalent to each other.

The postulation of initial to physics laws absolute time leads to curious consequences: laws can change over time. I.e. laws turn out to be not invariable, not so fundamentally basic for time determination, but they occur in time themselves. This is certainly an interesting approach that allows a fresh look at the evolution of the Universe. But this approach leaves the essence of time unexplained. On the contrary, the assumption of changing the laws of physics over time requires the mechanism of this change to be explained, so there must be some principle of the laws evolution over time. There is also another possibility: to say that time is ultimate and it exists for no particular reason, but it is a cognitive dead-end.

Thus, Shape Dynamics is trying to solve the problem of choosing initial conditions by introducing asymmetric solutions in time. Time exists, it is unchangeable, but things change over time, including the laws of physics (perhaps, new laws appear).

Another important feature of Shape Dynamics (which makes it even possible to reconstruct events in the past) is that it is consistent with the theory of hidden variables in quantum mechanics, i.e. with the idea that all particles have a position and velocity at any point in time (such an assumption suggests the need for a distinguished observer). This is exactly what Einstein has demanded from the theory and what became possible only under the dual description of his theory.

5. Conclusion

Alongside with the growth of scientific knowledge the intellectual intuition adapts to the formulation of new concepts and modernization of old ones (inverse relation is also true). With the incipiency of new physical theories (experimental confirmation is optional here) an updating of traditional question formulation is often required. For example, the theory of relativity introduces the relativity of simultaneity concept – a fundamentally new step in science, which requires a rethinking of the category of time (that ends up in GRT with eliminating of time). Another example is the quantum field theory, where the time as the distinction of past and future occurs only during the transition from micro- to macrolevel. The holographic principle, which has grown out of the possibility of a dual description of physical systems, offers again a completely new way of time understanding: time as projection (or requires the acceptance of two time origins). The multiverse (and the cyclic Universe) concepts raise a fundamental question on global time – whether in each universe its separate time exists or whether time is common to all of them.

Finally, the question of time dimension is a really new issue in the problem. Time has traditionally been considered as either a circle (cyclical) or as an arrow. If time can be of great dimension, as the folded spaces in superstring theory, it is most likely confirming the validity of the concept of time geometrization and shows a lack of grounds for search of essentially temporary categories. I.e., it is possible that time is a variety of space.

As it has become possible to find out, most of related to time issues have not been significantly changed, comparing to earlier attempts of its philosophical interpretation.

Considering that, the key stated at the beginning question – if the time exists – has no positive solutions. In those theories, where the answer is positive, it is fundamental in the sense that it is initial. This option cannot still define that the time is, but on the contrary, takes a step back in an attempt to answer this question. After declaring something as initial, we can continue considering it only in the spirit of negative theology, since nothing becomes the cause of it.

In the concepts, where time appears, it is possible to consider it existing; but so far the only variant for its explanation is the space. However, understanding of time (not at all new) as a movement in space (the sequence of events in space or even a special kind of space) sort of eliminates the time from the concept of time. These approaches deny their own temporal time specifics (which may become true).

It is likely that the part of the problem lies in the inability of intellectual intuition to exceed the bounds of the ordinary idea of time and the longing for fitting it into the familiar pattern of macrocosm. In this case, an effective way to overcome this would be a formulation of new concepts of time and space on the basis of the experimental results and mathematical description, which severs the tradition of thinking that produced formulated by Zeno problems. In a sense, the essence of his paradoxes specifically points to a disparity between the intellectual intuition and physical reality, rather than to the impossibility of movement.

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Notes

1. This refers not to the fact that the coordinate system is an objective description of space. Concerning objects and phenomena of microcosm the usual concept of distance loses its meaning, but the system of coordinates can be used as a convenient tool.
2. This section covers only amenable to physical interpretation ideas.
3. Descartes, who was Leibniz's predecessor in asserting the relativity of motion, could not, however, draw a conclusion about the relativity of time from that, and Leibniz, asserting the relativity, failed to notice the possibility of the relativity of simultaneity.
4. Interestingly, the assertion of the subjectivity of time (and, more widely, solipsism) does not deny the program of scientific research. In any case, the question remains: why we do we perceive the time in one way, but not another? Why do we imagine (in the case of solipsism) the world as we do, but not otherwise? The laws of physics should be the answer, determining our perception and thinking. Which means, that science is a search for what stands behind the phenomena, whatever they may be.
5. Hereinafter, it is assumed that if something is counter-intuitive, it does not lead to impossibility of something. This only means the inability to perceive the object of controversy at present.
6. If we accept this, it is fair to mention: non-locality exists, but only at the quantum level, as additional dimensions are folded and the macrocosm is local
7. There is no contradiction here. Einstein space is relative - it only means that its properties are determined by massive objects. But it exists and these objects are within this space. In a certain way, this space-time is an arena of actions, a background, just as Newton's space, but with a feedback.
8. There is a great temptation to rethink the theory of Plato's ideas in the light of the holographic principle.
9. Maldacena's universe is different from ours and, strictly speaking, his results are not applicable to our world. But that does not diminish his importance. The very possibility of a dual description, which allows us to understand complex theories with the help of relatively simple ones, is of great value.
10. This raises another important issue. Inside a black hole time and space reverse their roles, movement in time becomes movement in space and vice versa. The fact of the dual description in this case points to the fundamental indistinguishability of space and time.
11. They are unobservable, because the string math allows only closed strings (gravitons), unlike photons, to travel between the two universes.

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Logics for Physarum Chips

Andrew Schumann

University of Information Technology and Management in Rzeszow
Rzeszow, Poland

e-mail: andrew.schumann@gmail.com

Krzysztof Pancerz

University of Rzeszow,
Rzeszow, Poland

e-mail: kkpancerz@gmail.com

Abstract:
The paper considers main features of two groups of logics for biological devices, called *Physarum* Chips, based on the plasmodium. Let us recall that the plasmodium is a single cell with many diploid nuclei. It propagates networks by growing pseudopodia to connect scattered nutrients (pieces of food). As a result, we deal with a kind of computing. The first group of logics for *Physarum* Chips formalizes the plasmodium behaviour under conditions of nutrient-poor substrate. This group can be defined as standard storage modification machines. The second group of logics for *Physarum* Chips covers the plasmodium computing under conditions of nutrient-rich substrate. In this case the plasmodium behaves in a massively parallel manner and propagates in all possible directions. The logics of the second group are unconventional and deal with non-well-founded data such as infinite streams.
Keywords: Physarum polycephalum, unconventional computing, modern Voronoi diagram, p-adic valued logic, Petri nets.

1. Introduction

Physarum Chip is a biological sensing and computing device implemented in vegetative stage of acellular slime mould *Physarum polycephalum* [1]. Notice that *Physarum polycephalum* belongs to the species of order *Physarales*, subclass *Myxogastromycetidae*, class *Myxomycetes*, division *Myxostelida*. Plasmodium is its 'vegetative' phase represented as a single cell with a myriad of diploid nuclei. The *Physarum* Chips, designed in our project *Physarum Chip Project: Growing Computers From Slime Mould* supported by FP7, are programmed by spatio-temporal configurations of repelling and attracting gradients. There are several classes of *Physarum* Chips:

morphological processors, sensing devices, frequency-based, bio-molecular and microfluidic logical circuits, and electronic devices.

The *P. polycephalum* plasmodium behaves and moves as a giant amoeba. Typically, the plasmodium forms a network of protoplasmic tubes connecting the masses of protoplasm at the food sources which has been shown to be efficient in terms of network length and resilience. In the project we have proposed several logical methods for designing the *Physarum* Chips: conventional as well as unconventional. In this paper, we have generalized our results obtained for the whole period of the three-year project [1] – [42].

In Section II we consider conventional logics which are applicable for the case of nutrient-poor substrate of plasmodium feeding. In Section III we examine unconventional logics which can program the plasmodium behaviour in the case of nutrient-rich substrate. In Section IV we enumerate mathematical tools used in our formalization of *Physarum* computing. In Section V we propose a game-theoretic interface for *Physarum* computing.

2. Computing on Nutrient-Poor Substrate

2.1. Storage Modification Machines and Object-oriented Programming Language

Let us assume that the plasmodium feeds under the condition of *nutrient-poor substrate*. In this case the plasmodium can distinguish all attractants and repellents, and, as a result, those attractants and repellents involved in the stimulation of plasmodium gives a topology which can be defined as a *Voronoi diagram* [10], [19]. Within one Voronoi cell a reagent has a full power to attract or repel the plasmodium. In other words, within this cell the reagent determines the plasmodium behaviour completely. The distance for the cell is defined by intensity of reagent spreading like in other chemical reactions simulated by Voronoi diagrams. A reagent attracts or repels the plasmodium and the distance on that it is possible corresponds to the elements of a given planar set. When two spreading wave fronts of two reagents meet, this means that on the board of meeting the plasmodium cannot choose its one further direction and splits. Within the same Voronoi cell two active zones will fuse.

Now, suppose that the plasmodium feeds under the condition of *nutrient-rich substrate*. This means that for the plasmodium there is an information noise and the plasmodium cannot define where precisely attractants and repellents are located indeed.

Hence, in the case of nutrient-poor substrate with well distinguished localizations of attractants and repellents we can fully manage the plasmodium behaviour and propose a biological version of *storage modification machines*. These machines are defined in a new *object-oriented programming language* designed by us for *Physarum polycephalum* computing (OPL-Ph), [28], [29], [39], [42]. Within this language we can check possibilities of practical implementations of storage modification machines on plasmodia and their applications to behavioural science such as behavioural economics [31] and game theory [8], [30]. The proposed OPL-Ph can be used for developing programs for *P. polycephalum* by the spatial configuration of stationary nodes. Geometrical distribution of stimuli can be identified with a low-level programming language for *Physarum* machines.

2.2. Programmable Logic Controllers

At the beginning, we have proposed to construct logic gates through the proper geometrical distribution of stimuli for *P. polycephalum*. This approach has been adopted from the ladder diagram language widely used to program *Programmable Logic Controllers*. Flowing power has been replaced with propagation of plasmodium of *P. polycephalum*. Plasmodium propagation is stimulated by attractants and repellents. Rungs of the ladder can consist of serial or parallel connected paths of *Physarum* propagation. A kind of connection depends on the arrangement of regions of influences of individual stimuli. If both stimuli influence *Physarum*, we obtain alternative paths for its propagation. It corresponds to a parallel connection (i.e., the OR gate). If the stimuli influence *Physarum* sequentially, at the beginning only the first one, then the second one, we obtain a serial connection (i.e., the AND gate). The NOT gate is imitated by the repellent avoiding *Physarum* propagation.

In the proposed approach, we have assumed that each attractant (repellent) is characterized by its region of influence in the form of a circle surrounding the location point of the attractant (repellent), i.e., its center point. The intensity determining the force of attracting (repelling) decreases as the distance from it increases. A radius of the circle can be set assuming some threshold value of the force. The plasmodium must occur in a proper region to be influenced by a given stimulus. This region is determined by the radius depending on the intensity of the stimulus. Controlling the plasmodium propagation is realised by activating/deactivating stimuli.

Logic values for inputs have the following meaning in terms of states of stimuli: 0 – attractant/repellent deactivated, 1 – attractant/repellent activated. Logic values for outputs have the following meaning in terms of states of stimuli: 0 – absence of *P. polycephalum* at the attractant, 1 – presence of *P. polycephalum* at the attractant.

2.3.Petri Nets

At the second stage, we have adopted more abstract models than distribution of stimuli to program *P. polycephalum* machines which can be identified with programming in the high-level language. The choice fell on *Petri nets*. Petri nets were first developed by C.A. Petri. Petri nets are a powerful graphical language for describing processes in digital hardware. We have shown how to build Petri net models, and next implement as *P. polycephalum* machines, of basic logic gates AND, OR, NOT, and simple combination circuits [25], [29]. In our approach, we use Petri nets with inhibitor arcs. Inhibitor arcs are used to disable transitions. Inhibitor arcs test the absence of tokens in a place. A transition can only be if all its places connected through inhibitor arcs are empty. This ability of Petri nets with inhibitor arcs is used to model behaviour of repellents. Plasmodium of *Physarum* avoids light and some thermo- and salt-based conditions and this fact can be modelled by inhibitor arcs. The Petri net model (code in the high-level language) can be translated into the code in the low-level language, i.e., geometrical distribution of attractants and repellents of the *Physarum* machine.

In the proposed Petri net models, we can distinguish three kinds of places:

(1) Places representing *P. polycephalum*. For such a place, the presence of the token means that plasmodium of *Physarum* is present in the origin point. Otherwise, the absence of the token means that there is no plasmodium.

(2) Places representing input attractants or repellents. For such a place, the presence of the token means that the attractant/repellent is activated. Otherwise, the absence of the token means that attractant/repellent is deactivated.

(3) Places representing output attractants. For such a place, the presence of the token denotes the present of *Physarum* at the attractant (*Physarum* occupies the attractant). Otherwise, the absence of the token denotes the absence of *Physarum polycephalum* at the attractant.

In the *AND gate*, the transitions *T* represents the flow (propagation) of plasmodium from the origin place to the output attractant. *T* is enabled to fire if both attractants are activated, Fig.1.

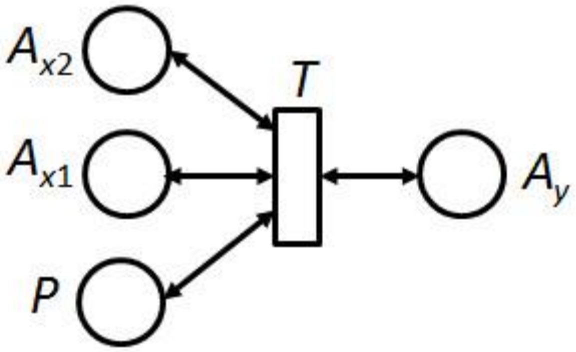

Fig. 1. AND gate

In the *OR gate*, the transitions T_1 and T_2 represent the alternative flows of plasmodium from the origin place to the output attractant. T_1 is enabled to fire if the first attractant is activated. T_2 is enabled to fire if the second attractant is activated, Fig.2.

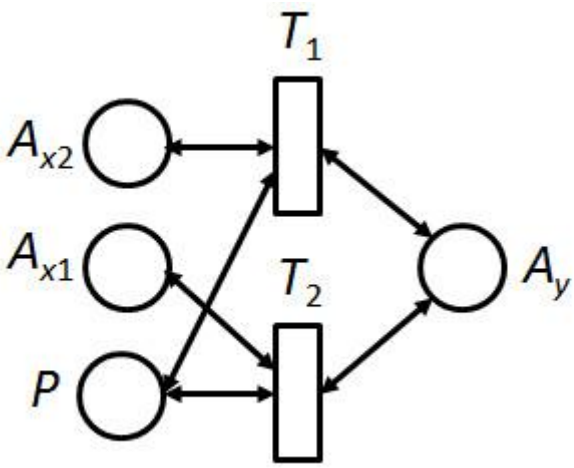

Fig. 2. OR gate

In the *NOT gate*, the transition T represents the flow (propagation) of plasmodium from the origin place to the output attractant. T is enabled to fire if the repellent is deactivated, Fig.3.

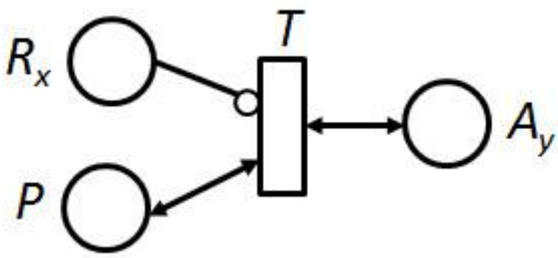

Fig. 3. NOT gate

Then, experimentally, we have built a *P. polycephalum demultiplexer* based on the ladder diagram structure. A demultiplexer is a device taking a single input signal and selecting one of many data-output lines, which is connected to the single input. So, using the ladder diagram approach, we can determine geometrical distribution of attractants and repellents for the 1-to-2 demultiplexer [32].

Petri nets are also a powerful tool for modelling concurrent systems. Moreover, Petri nets with inhibitor arcs can be used to model semantics of other types of computations, for example, pi-calculus or process algebra.

2.4.Concurrent Games

In our OPL-Ph we have analysed biological mechanisms for *Physarum* decision making and reconstructed the so-called *concurrent games* of plasmodia [30]. So, attractants may be regarded as *payoffs* for *Physarum*. Plasmodia occupy attractants step by step. By different localizations of attractants we can affect on *Physarum* motions differently. We can interpret the stimuli for *Physarum* motions as Boolean functions on payoffs. Boolean functions are designed by logical gates mentioned above (Fig.1–3).

In this way we have designed the zero-sum game between plasmodia of *Physarum polycephalum* and *Badhamia utricularis,* the so-called *PHY game* [15], [18], [23], [30], [33], [38], [40]. To simulate *Physarum* games, we have created a specialized software tool. The tool works under the client-server paradigm. The server window contains:

- a text area with information about actions undertaken,
- a combox for selecting one of two defined situations,
- start and stop server buttons.

Communication between clients and the server is realized through text messages containing statements of OPL-Ph. The locations of attractants and repellents are determined by the players during the game. At the beginning, origin points of *Physarum polycephalum* and *Badhamia utricularis* are scattered randomly on the plane. During the game, players can place stimuli. New veins of plasmodia are created. The server sends to clients information about the current configuration of the *Physarum* machine (localization of origin points of *Physarum polycephalum* and *Badhamia utricularis*, localization of stimuli as well as a list of edges, corresponding to veins of plasmodia, between active points).

So, we have proposed a game-theoretic visualization of morphological dynamics with non-symbolic interfaces between living objects and humans. These non-symbolic interfaces are more general than just sonification and have a game-theoretic form. The user interface for this game is designed on the basis of the following game steps: first, the system of OPL-Ph generates locations of attractants and repellents; second, we can chose *n* plasmodia/agents of *Physarum polycephalum* and *m* plasmodia/agents of *Badhamia utricularis*; third, we obtain the task, for example to reach as many as possible attractants or to construct the longest path consisting of occupied attractants, etc.; fourth, we can choose initial points for *Physarum polycephalum* transitions and initial points for *Badhamia utricularis* transitions; fifth, we start to move step by step; sixth, we define who wins, either *Physarum polycephalum* or *Badhamia utricularis*.

Strategies in games between *Physarum polycephalum* and *Badhamia utricularis* are described by rough sets defined on transition systems [26], [38], [40].

We claim that bio-inspired games might wake new interests in designing new games and new game platforms [33].

Thus, *Physarum* computing on nutrient-poor substrate can be considered storage modification machines and these machines are well formalizable in OPL-Ph. In this language we can define Petri nets and concurrent games. This language can be applied as programming language for *Physarum* logic circuits on nutrient-poor substrate. Hence, the programming language for *Physarum* behaviour we have constructed, on the one hand, simulates the *Physarum* behaviour and, on the other hand, shows which mathematical tools can be implemented in its behaviour.

3. Computing on Nutrient-Rich Substrate

3.1.Non-Aristotelian Extensions of Storage Modification Machines

Notice that under the conditions of nutrient-rich substrate, storage modification machines on plasmodia cannot be conventional. The point is that under these conditions we cannot approximate elementary (atomic) acts, i.e. we deal with a massive-parallel behaviour of plasmodia (they are to be expanded in all possible directions) [6], [17], [20]. Hence, we must extend our OPL-Ph by some unconventional tools to make OPL-Ph applicable for the case of nutrient-rich substrate, also.

For fixing *Physarum* computing on nutrient-rich substrate, we have constructed a *non-Aristotelian syllogistic* (*performative syllogistic*) [9], [10], [15], [19], [36] that models *Physarum* simultaneous propagations in all directions (i.e. it is massive-parallel). This system can logically simulate a massive-parallel behaviour in the propagation of any swarm. In particular, this system simulates the behaviour of collectives of Trematode larvae (miracidia and cercariae) [36]. Also, this syllogistic system of propagation can be used as basic logical theory for quantum logic (without logical atoms) [21]. In this theory we can build non-well-founded trees for which there cannot be logical atoms. This theory is much more expressive than standard spatial algorithms in simulating the plasmodium motions. We can define some unconventional algorithms on non-well-founded trees to make calculations on plasmodia more effective. These algorithms are implemented on plasmodia by means of reversible logic gates [3], [14].

Notice that the Aristotelian syllogistic is implementable for *Physarum* computing on nutrient-poor substrate, while performative syllogistic is applicable for *Physarum* computing on nutrient-rich substrate [9], [10], [15], [19], [36]. Performative syllogistic is an extension of Aristotelian one and includes the latter as its own part.

3.2.Reflexive Games

We have constructed an unconventional logic, which deals with non-well-founded data, namely with infinite streams and wave sets (sets of mutually defined infinite streams) [8], [17], [30]. Within this logic we can logically combine cellular automata, where Boolean functions are regarded as local transition rules which can change at in time. This symbolic logic can simulate the *Physarum* propagations with much localization of plasmodia.

Within the unconventional logic, we have introduced the notion of *payoff cellular automata* [8] instead of payoff matrices. By using these automata we can formalize *context-based reflexive games* for *k* players on different finite or infinite levels of reflexion. We have defined games as interactions among rational players, where decisions impact the payoffs of others, but players are limited by contexts that permanently change. A game is described by (i) its players who are presented in appropriate *transition rules of cellular automata*; (ii) players' possible strategies which are supposed known before the game and a combination of all possible payoffs from each strategy outcome gives the resulting payoffs which are collected as a *set of states of cellular automata*; (iii) a *neighbourhood of cellular automata* that makes some strategies actual and others non-actual (i.e. accepts the most important strategies in the given context at time *t*) and also changes or correct strategies. So, in this form of game description, players analyze strategies not purely logically, but contextually. Therefore players take decisions not only in an environment given by the payoff that corresponds to each possible outcome, but also in an environment of different other circumstances, e.g. by defining: which strategies can be accepted in this context, how they can be changed by the given context, how past contexts have influenced present contexts, whether some public announcements are false in fact, etc.

We have disproven the Aumann's agreement theorem (the so-called reflexion disagreement theorem) [11], [13]. This result can be obtained if (1) we assume that rational agents can become unpredictable for each other and try to manipulate; (2) we define probabilities on streams (e.g. on hypernumbers or p-adic numbers); (3) games are presented as coalgebras. This new theorem is an important statement within the new mathematics (coalgebras, transition systems, process calculi, etc.) which has been involved into game theory recently. Instead of the agreement theorem, the reflexion disagreement theorem is valid if we cannot obtain inductive sets, e.g. in case of sets of streams. The reflexion disagreement theorem opens the door for new mathematics in game theory and decision theory; in particular it shows that it has sense to use stream calculus, non-Archimedean mathematics, and p-adic analysis there. Within this mathematics we can formalize reflexive games of different reflexive levels (up to the infinite reflexive level). And these results are used by us for formalizing the game theory of plasmodia. Richer substrate due nutrients, more reflexive plasmodium game.

3.3.Theories on Hybrid Actions

We have defined an extension of process algebra, where simple actions of labelled transition systems cannot be atomic; consequently, their compositions cannot be inductive [8]. Their informal meaning is that in one simple action we can suppose the maximum of its modifications. Such actions are called *hybrid*. Then we propose two formal theories on hybrid actions (the hybrid actions are defined there as non-well-founded terms and non-well-founded formulas): group theory and Boolean algebra.

The group theory proposed by us can be used as the new design method to construct reversible logic gates on plasmodia [5], [14]. In this way, we should appeal to the so-called *non-linear permutation groups* [5], [8]. These groups contain non-well-founded objects such as infinite streams and their families. The theory of non-linear permutation groups proposed by us can be used for designing reversible logic gates on any behavioural systems. The simple versions of these gates are represented by logic circuits constructed on the basis of the performative syllogistic. It seems to be natural for behavioural systems and these circuits have very high accuracy in implementing. Our general motivation in designing logic circuits in behavioural systems without repellents is as follows: in this way, we can present behavioural systems as a calculation process more naturally; we can design devices, where there are much more outputs than inputs, for performing massive-parallel computations in the bio-inspired way; we can obtain unconventional (co)algorithms by programming behavioural systems. Computations on protoplasmic tree are understood as a kind of extension of concurrent processes defined in concurrent games. This extension is called context-based processes and they are defined in the theory of context-based games proposed by us.

Richer substrate due nutrients, more hybrid plasmodium action.

3.4.Neural Properties of Slime Mould and Modal Logics

There are two main properties of neural networks: lateral activation and lateral inhibition. The same properties are observed in *Physarum polycephalum* networks. We have generalized our studies of lateral inhibition effects in *P. polycephalum* behaviour in the way of constructing new syllogistics and modal logics. So, we have shown that there are two main possibilities of pairwise comparisons analysis in computer science: first, pairwise comparisons within a lattice, in this case these comparisons can be measurable by numbers (this one corresponds to lateral inhibition); second, comparisons beyond any lattice, in this case these comparisons cannot be measurable in principle (this one corresponds to lateral activation of neural networks). We have shown that the first approach to pairwise comparisons analysis is based on the conventional square of opposition and its generalization, but the second approach is based on unconventional squares of opposition [22].

Furthermore, the first approach corresponds to lateral inhibition in transmission signals and the second approach corresponds to lateral activation in transmission signals. For combining lateral inhibition and lateral activation in the same behaviour we introduced the notion of the so-called context based games to describe rationality of the slime mould. In these games we assume that, first, strategies can change permanently, second, players cannot be defined as individuals performing just one action at each time step. They can perform many actions simultaneously.

Under the conditions of nutrient-rich substrate the plasmodium has lateral activation effects and under the conditions of nutrient-poor substrate the plasmodium has lateral inhibition effects. We have shown that modal logic **K** corresponds to the lateral inhibition property and we can construct new modal logics, alternative to **K**, for embodying the lateral activation property [22].

Thus, we have extended our OPL-Ph by adding new tools: performative syllogistic, reflexive games, theories on hybrid actions, and modal logics for lateral activation, which allow us to simulate *Physarum* computing on nutrient-rich substrate.

4. Formalisation of Storage Modification Machines

In designing logics in OPL-Ph for *Physarum* Chips we have used the following mathematical tools:

4.1.Process Algebra

In OPL-Ph we have been based on *process-algebraic* formalizations of *Physarum* storage modification machines [34]. So, we have considered some instructions in *Physarum* machines in terms of process algebra like: add node, remove node, add edge, remove edge. Adding and removing nodes can be implemented through activation and deactivation of attractants, respectively. Adding and removing edges can be implemented by means of repellents put in proper places in the space. An activated repellent can avoid a plasmodium transition between attractants. Adding and removing edges can change dynamically over time. To model such behaviour, we have proposed a high-level model, based on timed transition systems [26], [27].

In this model we have defined the following four basic forms of *Physarum* transitions (motions): direct (direction: a movement from one point, where the plasmodium is located, towards another point, where there is a neighbouring attractant), fuse (fusion of two plasmodia at the point, where they meet the same attractant), split (splitting plasmodium from one active point into two active points, where two neighbouring attractants with a similar power of intensity are located), and repel (repelling of plasmodium or inaction).

In *Physarum* motions, we can perceive some ambiguity influencing on exact anticipation of states of *Physarum* machines in time. In case of splitting plasmodium, there is some uncertainty in determining next active points (attractants occupied by plasmodium), if a given active point is known. This uncertainty does not occur in case of direction, where the next active point is uniquely determined. To model ambiguity in anticipation of states of *Physarum* machines, we propose to use rough set theory [26], [38], [40], [41]. Analogously to the lower and upper approximations, we define the lower and upper predecessor anticipations of states in the *Physarum* machine. Behaviour of *Physarum* machines can also be modelled using Bayesian networks with probabilities defined on rough sets [41].

Thus, we have proposed some timed and probabilistic extensions of standard process algebra to implement timed and rough set models of behaviour of *Physarum* machines in OPL-Ph.

4.2.Computation on Trees

Computations on tree are usually represented by spatial algorithms like Kolmogorov-Uspensky machines. Theoretically, Turing machines, Kolmogorov-Uspensky machines, Schönhage's storage modification machines, and random-access machines have the same expressibility power. In other words, the class of functions computable by these machines is the same. Unfortunately, the computational power of their implementations on the *Physarum polycephalum* medium is low.

The point is that not every computable function can be simulated by plasmodium behaviours: first, the motion of plasmodia is too slow (several days are needed to compute simple functions such as 5-bit conjunction, 3-bit adder, etc., but the plasmodium stage of *Physarum polycephalum* is time-limited, therefore there is not enough time for realizations, e.g., of thousands-bit functions); second, the more attractants or repellents are involved in designing computable functions, the less accuracy of their implementation is, because the plasmodium tries to be propagated in all possible directions and we will deal with indirected graphs and other problems; third, the plasmodium is an adaptive organism that is very sensitive to environments, therefore it is very difficult to organize the same laboratory conditions for calculating the same *k*-bit functions, where *k* is large.

To make computations on tree more expressive we have proposed the performative syllogistic – syllogistic system of propagation [9], [10], [15], [19]. This system can logically simulate a massive-parallel behaviour in the propagation of collectives of parasites [36]. So, protoplasmic trees can be interpreted as syllogistic trees. In this way while Aristotelian syllogistic may describe concrete directions of *Physarum* spatial expansions, performative syllogistic proposed by us may describe *Physarum* simultaneous propagations in all directions. Therefore, while for the implementation of Aristotelian syllogistic we need repellents to avoid some possibilities in the *Physarum* propagations, for the implementation of performative syllogistic we do not need them. Performative syllogistic has a p-adic valued semantics and satisfies p-adic valued probabilities [6], [15]. The syllogistic can be extended to a more general theory of context-based games. This theory is proposed in [8]. Within this theory we can define algorithms for computing on protoplasmic tree.

Computations on protoplasmic tree are understood as a kind of extension of concurrent processes defined in concurrent games proposed by Samson Abramsky [30]. This extension is called context-based processes and they are defined in the theory of *context-based games* proposed by us.

Thus, we have defined some unconventional algorithms on non-well-founded trees to make calculations on plasmodia more expressive. These algorithms can be used for constructing an alternative quantum logic (without logical atoms) [21] and a simulation model for propagating parasites [35].

4.3.Cellular Automata

The universe, where *Physarum* lives, consists of cells possessing different topological properties according to the intensity of chemo-attractants and chemo-repellents. The intensity entails the natural or geographical neighbourhood of the set's elements in accordance with the spreading of attractants or repellents. As a result, we obtain Voronoi cells. In this structure we can implement cellular automata [19]. Taking into account the fact that the plasmodium is very sensitive to the environment and can change its strategies we can extent the standard notion of cellular automata and assume that transition rules can change in an appropriate neighbourhood in accordance with some outer conditions at time step $t = 0, 1, 2...$ The theory of these automata with changing transition rules is proposed in [8], [17].

The plasmodium implements *cellular automata with changing transition rules*. Within these automata we can define context-based concurrent formal theories [5], [14], [17].

4.4.p-Adic Logic and Arithmetics

The slime mould is considered a natural fuzzy processor with fuzzy values on the set of p-adic integers [6], [7], [20], [24]. The point is that in any experiment with the slime mould we deal with attractants which can be placed differently to obtain different topologies and to induce different transitions of the slime mould. If the set A of attractants, involved into the experiment, has the cardinality number $p - 1$, then any subset of A can be regarded as a condition for the experiment such as "Attractants occupied by the plasmodium". These conditions change during the time, $t = 0, 1, 2, \ldots$, and for the infinite time, we obtain p-adic integers as values of fuzzy (probability) measures defined on conditions (properties) of the experiment. This space is a semantics for p-adic valued fuzzy syllogistics we constructed for describing the propagation of the slime mould [6]. This syllogistics can be extended to a *p-adic valued logic* and *p-adic valued arithmetics* [7], [20], [24]. Within this logic we can develop a context-based game theory [8], [30]. All these logical tools can be implemented on plasmodia by conventional and unconventional reversible logic gates [3], [14].

We have proposed to use p-adic valued fuzziness and probabilities for measuring behaviours which cannot be measured additively [11], [13]. Then we have constructed a natural deductive system for describing all possible experiments with *P. polycephalum* [7]. This system is p-adic many valued. We have considered possibilities for applying p-adic valued logic **BL** to the task of designing the *Physarum* Chips. If it is assumed that at any time step t of propagation the slime mould can discover and reach not more than $p - 1$ attractants, then this behaviour can be coded in terms of p-adic numbers. As a result, this behaviour implements some p-adic valued arithmetic circuits and can verify p-adic valued logical propositions.

We have offered two unconventional arithmetic circuits: adder and subtractor defined on finite p-adic integers [24]. Adder and subtractor are designed by means of spatial configurations of several attractants and repellents which are stimuli for the plasmodium behaviour. As a result, the plasmodium could form a network of protoplasmic veins connecting attractants and original points of the plasmodium. Occupying new attractants is considered in the way of adders and leaving some attractants because of repelling is considered in the way of subtracters. On the basis of p-adic adders and subtractors we can design complex p-adic valued arithmetic circuits within a p-adic valued logic proposed by us.

So, p-adic valued logic and p-adic valued arithmetic are implementable on plasmodia. In the meanwhile, *Physarum* computing on nutrient-poor substrate is expressible by finite p-adic integers and *Physarum* computing on nutrient-rich substrate is expressible by infinite p-adic integers.

4.5.Non-well-founded Formal Theories

We have proposed two *formal theories on hybrid actions* (non-well-founded data): group theory and Boolean algebra [17]. Both theories possess many unusual properties such as the following one: the same member of this group theory behaves as multiplicative zero in respect to some members and as multiplicative unit in respect to other members. This group theory is used to design reversible logic gates on plasmodia,

Hence, to formalize *Physarum* computing we have proposed new formal theories such as theory of non-linear permutation groups to design unconventional reversible logic gates [5], [14].

4.6.Reversible Logic Gates

We have considered different ways of designing reversible logic gates on *P. polycephalum* motions using controlling stimuli such as attractants and repellents [3], [5], [14]. Repellents are needed because of uncertainty in the direction of plasmodium propagation to eliminate some directions as unimportant. In this way, we can construct conventional reversible logic gates: the CNOT gate, the FREDKIN gate, the TOFFOLI gate, etc. Combinations of reversible logic gates are regarded as matrix multiplications. Nevertheless, the plasmodium in its networking can permanently change its decisions and without repellents we have an extension of matrix multiplication group theory. Within this extension we can design unconventional reversible logic gates, where the number of inputs and outputs is uncertain. For designing logic gates we have proposed to use Petri net models that can be treated as a high level description. Petri net models enable us to reflect propagation of protoplasmic veins of the plasmodium in consecutive time instants (step by step).

4.7.Actin Filament Networks

Plasmodia consist mainly of actin filaments. We have proposed *artificial actin filament networks* where inputs are different stresses and outputs are formations and destructions of filaments, on the one hand, and as assemblies and disassemblies of actin filament networks, on the other hand [16]. Hence, under different external conditions we observe dynamic changes in the length of actin filaments and in the outlook of filament networks. As we see, the main difference of actin filament networks from others including neural networks is that the topology of actin filament networks changes in responses to dynamics of external stimuli. Some new filaments/processors can appear in one conditions and they can disappear in other conditions.

4.8.Swarm Computing

By modelling the plasmodium behaviour in the *Physarum* Chips we can simulate some patterns of collective intelligent behaviours of animal or insect groups: flocks of birds, colonies of ants, schools of fish, swarms of bees, etc. for which there are ever emergent patterns which cannot be reduced to a linear composition of elementary subsystems properly [2], [4], [31]. In swarm intelligence the Travelling Salesman Problem can be solved: more shorter distance between cities (pieces of food for the plasmodium), more attracting they are, as well as the Generalized Assignment Problem can be solved: the tubes of the plasmodium are regarded as agents, the nutrient sources as tasks, the amount of nutrient as profit, and the distance as cost. We show that by using p-adic integers we can code different emergent patterns so that the implementation of some unconventional algorithms of p-adic arithmetics and logic can be more applicable than conventional automata.

5. Game-Theoretic Interface for Storage Modification Machine

5.1.Chemical Interface

In moving, the plasmodium switches its direction or even multiplies in accordance with different bio signals attracting or repelling its motions, e.g. in accordance with pheromones of bacterial food, which attract the plasmodium, and high salt concentrations, which repel it. So, the plasmodium motions can be controlled by different topologies of attractants and repellents so that the plasmodium can be considered a programmable biological device in the form of a timed transition system, where attractants and repellents determine the set of all plasmodium transitions. Furthermore, we can define p-adic probabilities on these transitions and, using them, we can define a knowledge state of plasmodium and its game strategy in occupying attractants as payoffs for the plasmodium.

We can regard the task of controlling the plasmodium motions as a game and we can design different interfaces in a game-theoretic setting for the controllers of plasmodium transitions by chemical signals [23].

5.2.Bio-inspired Game Theory

We have proposed a bio-inspired game theory on plasmodia, i.e. an experimental game theory, where, on the one hand, all basic definitions are verified in the experiments with *Physarum polycephalum* and *Badhamia utricularis* and, on the other hand, all basic algorithms are implemented in OPL-Ph [15], [18], [23], [30], [33], [38], [40]. Our results allow us to claim that the slime mould can be a model for concurrent games and context based games. In context based games, players can move concurrently as well as in concurrent games, but the set of actions is ever infinite. In our experiments, we follow the following interpretations of basic entities:

- Attractants as payoffs;
- Attractants occupied by the plasmodium as states of the game;
- Active zones of plasmodium as players;
- Logic gates for behaviours as moves (available actions) for the players;
- Propagation of the plasmodium as the transition table which associates, with a given set of states and a given move of the players, the set of states resulting from that move.

In the *Physarum* game theory we can demonstrate creativity of primitive biological substrates of plasmodia. The point is that plasmodia do not strictly follow spatial algorithms like Kolmogorov-Uspensky machines, but perform many additional actions. So, the plasmodium behaviour can be formalized within strong extensions of spatial algorithms, e.g. within concurrent games or context-based games.

5.3.Go Games

In the universe of 5-adic integers, we have simulated the motions of *P. polycephalum* plasmodium by the game of Go [15], [18], [38]. We have considered two syllogistic systems implemented as Go games: the Aristotelian syllogistic and performative syllogistic. In the Aristotelian syllogistic, the locations of black and white stones are understood as locations of attractants and repellents, respectively. In the performative syllogistic, we consider the locations of black stones as locations of attractants occupied by plasmodia of *P. polycephalum* and the locations of white stones as locations of attractants occupied by plasmodia of *Badhamia utricularis*. The Aristotelian syllogistic version of Go game is a coalition game. The performative syllogistic version of Go game is an antagonistic game. We described selected functionality of the current version of a new software tool, called *PhysarumSoft*, developed for programming *Physarum* machines and simulating *Physarum* games. The tool was designed for the Java platform. We proposed a rough set approach for description of a strategy game created on the *Physarum* machine. The strategies of such a game are approximated on the basis of a rough set model, describing behavior of the *Physarum* machine, created according to the VPRSM (Variable Precision Rough Set Model) approach [38].

Thus, we have proposed game-theoretic interface for *Physarum* Chips.

Conclusion

We have built up two basic groups of logics for the *Physarum* Chips: the first ones deal with the plasmodium behaviour under the conditions of nutrient-poor substrate and the second one deal with the plasmodium behaviour under the conditions of nutrient-rich substrate. The first logics can be conventional (classical, multi-valued, modal, or fuzzy), while the second are strong extensions of the first logics and cannot be conventional and they engage the so-called non-well-founded data such as infinite streams.

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Continuous-Logical Methods in Mathematical Economics

Vitaly I. Levin

Mathematics Department,
Penza State Technological University,
Russia

e-mail: vilevin@mail.ru

Abstract:
An application of continuous logic for the mathematical description of economical systems is given. Parallel, sequential, parallel-sequential and sequential-parallel systems are calculated using continuous logic (CL) methods.
Keywords: continuous logic, logical determinant, mathematical economics.

1.

In 1978 the author for the first time indicated a possibility to use continuous logic (CL) for solution of the optimal tasks of mathematical economics. The application of CL is represented by operations of maximum and minimum. Also, there was detected a possibility to evaluate industrial systems with the help of CL of various metrics: speed, productivity, and the modes of operations of these systems determined by ratio 'more' or 'less' between temporary parameters of operations, executed there. Nevertheless, there were not developed methods of the CL-analysis and synthesis of the optimal schedules of execution of operations in systems for a long time. Nowadays CL-methods in mathematical economics represent an independent branch of this science, with the research methodology and significant results. In the given paper let us review some of these results.

2.

Let us consider a sequential system with m blocks executing m various operations. In the system, n jobs consisting of m indicated operations simultaneously occur. The operation execution i for the job j is given by the matrix $A=\|a_{ij}\|$. These jobs are started in the system and pass in the blocks $1,\dots,m$ in the same order $1,2,\dots,n$. Thus, each job passes in the next block i at once after an output from the previous block and release of block i from a prior operation. We assume that the speed of the given system is characterized by time of passing of all jobs through all blocks:

$$T = A^{\vee} = |a_{ij}|^{\vee}, \qquad (1)$$

where T is a disjunctive logical determinant (LD) $A^{\vee}$ from a matrix A of operations. The LD $|a_{ij}|^{\vee}$ is a function $\{a_{ij}\} \to a^r$, where a^r is the r-th element of a matrix A. The formula (1) reduces a calculation (analysis) of speed of the sequential system to a calculation (analysis) in the LD $A^{\vee}$. In the LD there is a CL function satisfying (1) which expresses time of operation a_{ij}. Due to T the speed of job (productivity) of the system is expressed by $v = n/T$.

3.

For a sequential system, the average load for the k-th block is defined thus:

$$R_k = \sum_{j=1}^{n} a_{kj} / T \tag{2}$$

and let $r_k(t)$ be an instant load at the arbitrary moment t, then the average load of the system and its instant load at the moment t are defined as follows:

$$R = \sum_{k=1}^{m} \sum_{j=1}^{n} a_{kj} / mT, \ r(t) = \sum_{k=1}^{m} r_k(t)/m \,. \tag{3}$$

Equations (1) – (3) for the calculation of characteristics of average load of the block and of the system show that first the LD $A^{\vee}$ from a matrix A of job time should be calculated. For finding the characteristics of instant load it is necessary to determine matrixes of moments of beginning of jobs in $\bar{T} = \|\bar{t}_{ij}\|$ blocks and ending of jobs in $\underline{T} = \|\underline{t}_{ij}\|$ blocks. Here $\underline{t}_{ij}$ ($\bar{t}_{ij}$) are moments of the beginning (ending) of the job j in the block i. Let $A_{rk}^{\vee}$ be a disjunctive LD from r first lines and k first columns of a matrix A and $A^* = \|A_{rk}^{\vee}\|$ be a matrix attached to A. We know that

$$\bar{T} = A^* . \tag{4}$$

It is clear that

$$\bar{T} - \underline{T} = A \,. \tag{5}$$

By calculating A^* with the help of wave algorithm, we can obtain $\bar{T}$ and then $\underline{T}$ from (5). Hence, a characteristic $r_k(t)$ of instant load of the block can be defined in the following manner:

$$r_k(t) = \begin{cases} 1, \ t \in \bigcup_{i=1}^{n} [\underline{t}_{ki}, \bar{t}_{ki}], \\ 0, \ \text{otherwise.} \end{cases} \tag{6}$$

4.

Let us consider a special class represented by sequential systems dependent on time by arrival of jobs, in which there is both order and moment of arrival of jobs in the system. If this order is $1,2,\ldots,n$, and an appropriate moment is $\tau_1, \tau_2, \ldots, \tau_n$, time of passing of all jobs through the system is af follows:

$$T = \begin{vmatrix} \tau_1 & \tau_2 - \tau_1 & \cdots & \tau_n - \tau_{n-1} \\ a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix}^{\vee} - \tau_1. \quad (7)$$

From (7) it is evident that any system of this class is equivalent to a sequential system by arrival of jobs having an extended matrix of job time.

5.

Let us concentrate now on a parallel system from m functional one-type blocks $1,2,\ldots,m$ for the execution of n one-type jobs $(n \geq m)$. The execution time of job j in the block i is given by a matrix $A = \|a_{ij}\|$. Jobs arrive in the system by the given order $P_n = (j_1,..., j_n)$. Thus, at the moment $t = 0$ the jobs $j_1,..., j_m$ boot in blocks $1,\ldots,m$, which begin jobs and arrive in the process of release by the consequent jobs $j_{m+1},..., j_n$. Any characteristics of speed of the given system are expressed by a vector $t = (t_1,..., t_n)$, where t_k is the moment of ending of job j_k. The execution time of all n jobs and speed of their execution (productivity) is defined thus:

$$T = \bigvee_{i=1}^{n} t_i, \qquad v = n/T, \quad (8)$$

where $\vee = \max$ is an operation of CL-disjunction. The vector t is calculated easily in the case of homogeneous system, where the blocks are identical on speed, so the matrix of job time A degenerates in a vector of job time $a = (a_1,..., a_n)$, where a_j is an execution time of job j in any block. Let $\tilde{a} = (\tilde{a}_1,..., \tilde{a}_n)$ be a vector distinguished from a vector a by rearrangements of elements according to the given order P_n of starting jobs. Then the moments t_k are connected by such a recurrence:

$$t_k = \begin{cases} \tilde{a}_k, \ 1 \leq k \leq m; \\ \begin{vmatrix} t_1 \\ \cdots \\ t_{k-1} \end{vmatrix}^{(k-m)} + \tilde{a}_k, \quad m+1 \leq k \leq n, \end{cases} \quad (9)$$

allowing sequentially to calculate these moments for the homogeneous system of jobs in the terms of CL, for serial LD of sort $\begin{vmatrix} a_1 \\ \cdots \\ a_N \end{vmatrix}^r$ in (9) expressed in these terms.

6.

Characteristics of load of the parallel system are defined as follows. Any block k of the parallel system begins to operate at the moment $t = 0$, gets the job 1, then at the moment $t_1(k)$ it gets the job 2, ends at the moment $t_2(k)$ ..., at last, at the moment $t(k)$ it ends the last job. From this, the expressions of average and instant load of the k-th block are defined in the following way:

$$R_k = t(k)/T, \quad r_k(t) = \begin{cases} 1, & 0 \leq t \leq t(k), \\ 0, & t > t(k), \end{cases} \Bigg| \ k = \overline{1,m}, \quad (10)$$

and expressions of average and instant load of the system:

$$R = [\sum_{k=1}^{m} t(k)]/mT, \quad r(t) = \begin{cases} 1, & t \le t^1, \\ 1-(i/m), & t^i \le t \le t^{i+1}, \ i = \overline{1, m-1}, \\ 0, & t > t^m. \end{cases} \tag{11}$$

where $\{t^i\}$ is a set $\{t(i)\}$, ordered by increase.

7.

A special class of parallel systems dependent on time by arrival of jobs, in which the arrivals of jobs are given by the order $P_n = (j_1,..., j_n)$ and the moment $\tau_1,..., \tau_n$ is obtained as follows. The calculations and analysis of such systems is based on a formula-analogue (9) (the case of the homogeneous system):

$$t_k = \begin{cases} \tau_k + \tilde{a}_k, & 1 \le k \le m, \\ \left(\tau_k + \begin{vmatrix} t_1 \\ \cdots \\ t_{k-1} \end{vmatrix}^{(k-m)} \right) + \tilde{a}_k, & m+1 \le k \le n. \end{cases} \tag{12}$$

8.

Let us consider a parallel-sequential system from M consisting of joint steps with m_i parallel one-type blocks of equal speed at the i-th step. At an input of the system, the sequence $P_{1n} = (j_{11},..., j_{1n})$ arrive from n jobs $1,\ldots,n$, passing through it by steps, where appropriate jobs are executed. The execution time of operation i for the job j is given by the matrix $A = \|a_{ij}\|$. The order of job execution inside each step is determined by laws of operation of parallel systems, and transition step by step (laws of operation of sequential systems). Thus, the order of jobs at an output of step does not coincide with the order at an input in general case. Let us designate a moment of the ending of the q-th job operation started at the k-th operation by t_{qk}. The characteristic of speed of the whole system is expressed by time T of execution of all jobs expressed by t_{qk} with the help of a disjunction of CL:

$$T = \bigvee_{k=1}^{n} t_{Mk}, \tag{13}$$

so the calculation of T is reduced to a calculation of a matrix $\|t_{qk}\|$. The latter is reduced to a recurrence, allowing to get these characteristics in the terms of CL:

$$t_{qk} = \begin{vmatrix} t_{q-1,1} \\ \cdots \\ t_{q-1,n} \end{vmatrix}^{(k)} + \tilde{a}_{qk}, \quad \text{while} \quad 1 \le k \le m_q;$$
$$t_{qk} = \left(\begin{vmatrix} t_{q-1,1} \\ \cdots \\ t_{q-1,n} \end{vmatrix}^{(k)} \vee \begin{vmatrix} t_{q1} \\ \cdots \\ t_{q,k-1} \end{vmatrix}^{(k-m_q)} \right) + \tilde{a}_{qk}, \text{ while } m_q + 1 \le k \le n. \quad (14)$$

In (14), the matrixes $\tilde{A} = \|\tilde{a}_{ij}\|$ are obtained from the matrix $A = \|a_{ij}\|$ of rearrangements of elements in each q-th line according to the order of job start at the q-th job (the order of ending at the $(q-1)$-th operation).

9.

Le us examine a consecutive-parallel system consisting of parallel joint branches as a sequence of blocks. In the k-th branch there are m_k blocks. Each k-th branch can execute any of n ($n \ge M$) jobs, submitted to the system, by splitting at m_k several sequential jobs executed in appropriate blocks of the branch. The speed of any k-th branch is given by a matrix $A(k) = \|a_{ij}(k)\|$, where $a_{ij}(k)$ is a time execution in the k-th branch of operation i for the job j. The order of job execution in each branch is defined by laws of operation of sequential systems by a sequence of jobs $P_n = (j_1,..., j_n)$ among branches (laws of job of parallel systems). The allocation of a sequence of jobs P_n among branches depends on the order of release of the first blocks of branches. These blocks derivate the parallel system with a matrix of times of jobs as follows:

$$A = \|a_{kj}\|, \ \ a_{kj} = a_{1j}(k), \ \ k = \overline{1,M}, \ \ \ j = \overline{1,n}. \quad (15)$$

Calculating this system allows us to find the order and moments of release of blocks and through them to find an allocation of an entry sequence of jobs P_n among branches and moments of arrival of these jobs in various branches. After that the calculation and analysis of the whole system is reduced to the same procedures with separate branches in a mode dependent on time of arrival of jobs. So, the time execution of all jobs in the k-th branch T_k, and the load R_k of the common execution time of all jobs in the system are defined thus:

$$T = \mathop{\vee}_{k=1}^{M} t_k, \quad R = \sum_{k=1}^{M} R_k / M. \quad (16)$$

10.

Let us return to the tasks of calculation and analysis of synthesis of the whole system. They consist in a choice of set of acceptable procedures of job execution of optimal procedure, when characteristics of the system have the best values. The task of synthesis of static system is simple. The parallel system with m blocks is intended for the execution of n jobs $(n \ge m)$. The time execution of job j in the block i is given by a matrix $A = \|a_{ij}\|$. The set of jobs W is executed by acceptable splitting into subsets $W_1,..., W_m$, executed in appropriate blocks. Total operating time of all blocks is:

$$D = \sum_{i=1}^{m} \sum_{j \in W_i} a_{ij} \quad . \tag{17}$$

It is required to select an optimal splitting of jobs into subsets W_i where $D = \min$. Two variants of this problem are possible: 1) without limitations by a cardinal number of subsets W_i; 2) with limitations of the sort: $b_i \le |W_i| \le c_i$. We see that

$$D_{\min} = A^{1\wedge} \text{ (in the first case); } D_{\min} = A^{2\wedge} \text{ (in the second case),} \tag{18}$$

Here $A^{1\wedge}$ and $A^{2\wedge}$ are conjunctive LD of the 1-st and 2-nd sort with limitations by the sums of elements from a time matrix A. By the definition of $A^{1\wedge}$ there is a function of sort $\wedge_q \sum_q^{/} a_{ij}$, where $\wedge = \min$ is a conjunction of CL and $\sum_q^{/} a_{ij}$ is a sum of elements a_{ij}, including only one element from each column of the matrix A. Further, $A^{2\wedge}$ is a function $\wedge_q \sum_q^{//} a_{ij}$, where $\sum_q^{//} a_{ij}$ is a sum of elements a_{ij}, including only one element from each column of the matrix A and p_i elements from the i-th line, where $b_i \le p_t \le c_i$. Thus, to solve this problem it is necessary to calculate an appropriate LD. Hence, the value of LD specifies the value of $D_{\min}$ and the optimal allocation of jobs in blocks (the presence of element a_{ij} in an expression of LD means an attachment of the j-th job to the i-th block). The representation of (18) also shows that the analysis of the optimal static system is reduced to the analysis of behaviour of appropriate LD with changes of elements. In (18), the LD is a CL-function from their elements, the value $D_{\min}$ expresses time a_{ij} in the terms of CL.

According to the simplified formula of calculation of LD, we have $A^{1\wedge} = \sum_j \wedge_k a_{kj}$ for an arbitrary k-th block of all those jobs, which execution time in this block is a minimum compared with other blocks.

11.

The problem of synthesis of the sequential system with m blocks which execute n jobs consists in searching the optimal order $P_{opt} = (j_1,..., j_n)$ of job execution with the execution time of all jobs $T = \min$. The solution of this task is most simple in the case of $m = 2$. Any two jobs i, j in P_{opt} following the order $i \to j$, the execution conditions satisfies:

$$a_{1i} \wedge a_{2j} \le a_{1j} \wedge a_{2i} \, , \tag{19}$$

where $\wedge = \min$ is a conjunction of CL. The condition of (19) enables to design simple deciding rules for finding the optimal order of jobs P_{opt} without searches. It is interesting that in the case of $m = 2$ the solution of the problem of synthesis is searched in the class of the permutation schedules, i.e. sequences of jobs. It is connected to the following fact: P_{opt} with $m = 2$ lies in the class of the permutation schedules. Thus, any two jobs i, j in P_{opt} following the order $i \to j$, the execution conditions are:

$$a_{1i} \wedge a_{2j} \le a_{1j} \wedge a_{2i}, \quad a_{2i} \wedge a_{3j} \le a_{2j} \wedge a_{3i},$$
$$(a_{1i} + a_{2i}) \wedge (a_{1i} + a_{3j}) \wedge (a_{2j} + a_{3j}) \le (a_{1j} + a_{2j}) \wedge (a_{1j} + a_{3i}) \wedge (a_{2i} + a_{3i}). \tag{20}$$

The search of P_{opt} in the system with $m = 3$ blocks is carried out in the way: 1) the construction of graph of priorities of jobs linking by an arc $i \to j$, which represents jobs i, j satisfying the condition of (20); 2) finding in the graph any hamiltonian path, which gives P_{opt}. The given algorithm of searching the order P_{opt} in the system with $m = 3$ blocks is more complex than solution rules for systems with $m = 2$ blocks.

12.

The sequence of jobs $P = (i_1,..., i_k, i, j,..., i_n)$ is strongly (poorly) separable, if the rearrangement of any pair of jobs i, j increases (decreases) a time moment of the ending of subsequence of jobs $(i_1,..., i_k, i, j)$ in all the blocks q (even in one block q), $q = \overline{2,m}$. Such a limitation practically allows to formulate in the terms of CL and LD some general analytical conditions of optimality about jobs in systems with any number m of blocks.

1) For the sequence $P = (i_1,..., i_k, i, j,..., i_n)$ of passing of n jobs through m blocks it is optimal (and strongly separable) that at any ordered pair (i, j) of adjacent jobs from P the time of execution of jobs satisfies the following conditions:

$$\left.\begin{array}{ll} A^{\vee}_{s,s+1}(i,j) \le A^{\vee}_{s,s+1}(j,i), & s = \overline{1,m-1}; \\ A^{\vee}_{s,s+2}(i,j) \le A^{\vee}_{s,s+2}(j,i), & s = \overline{1,m-2}; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ A^{\vee}_{1m}(i,j) \le A^{\vee}_{1m}(j,i), & \end{array}\right\}, \tag{21}$$

which appear in a special disjunctive logical determinant up to the i-th and j-th columns of the matrix $A = \|a_{ij}\|$ of job time.

$$A^{\vee}_{sr}(i,j) = \begin{vmatrix} a_{si} & a_{sj} \\ \cdots & \cdots \\ a_{ri} & a_{rj} \end{vmatrix}^{\vee}, \quad 1 \le s \le r \le m \; ; \tag{22}$$

2) For the sequence P it is optimal (poorly separable) that at any pair of adjacent jobs (i, j) the time of jobs satisfies the conditions with the same LD:

$$[A^{\vee}_{12}(i,j) \le A^{\vee}_{12}(j,i)] \cup [A^{\vee}_{s3}(i,j) \le A^{\vee}_{s3}(j,i), s = \overline{1,2}] \cup \ldots \cup [A^{\vee}_{sm}(i,j) \le A^{\vee}_{sm}(j,i), s = \overline{1,m-1}] \tag{23}$$

Since LD is a CL-function, the conditions of (21), (23) express jobs in the terms of CL. With $m = 2$ the conditions of (21) and (23) coincide to give (19), therefore, we have the necessary and sufficient condition of an optimality of systems with two blocks. With $m = 3$ the conditions of (21) is reduced to (20). With $m \ge 3$ the condition of (21) is harder than the conditions of (23). The search in systems with $m \ge 4$ blocks with the help of sufficient conditions of an optimality (21) is carried out as well as in the case of $m = 3$, with using the graph of priorities of jobs. The search in the systems with $m \ge 3$ blocks with the help of necessary conditions of an optimality (23) is carried out in the following manner: 1) the creation of graph of priorities of jobs linking by an arc $i \to j$, which represent jobs satisfying the condition of (23); 2) searching in the graph all hamiltonian paths giving a sequences P, suspicious on an optimality; 3) in P_{opt} it is selected P, for which $T = \min$.

13.

For sequential systems with large numbers of blocks m and jobs n, and also for systems of other construction (parallel and more complex), analytical conditions of an optimality do not work or in general are absent. Therefore, synthesis of such systems is usually carried out by a branch and bound algorythm. The efficiency of this method essentially depends on the force of used estimations of time T of execution of all n operations, common for all possible sequences of jobs of sort $P^r = (R^r, Q^r)$, where R^r is a fixed sequence of r first operations, and Q^r is a set of all possible sequences of others $n-r$ jobs. The application of CL and LD allows us to receive strong estimations of characteristics of systems. It is clear that the value of characteristic for an initial site R^r of sequence P^r, as shown above, is expressed precisely by operations of CL and LD, and estimation of characteristic for the rest site Q^r turns out by a choice of the best (worst) case, that uses operations of continuous logic $\vee = \max$ and $\wedge = \min$. So, for the parallel system with m blocks and the matrix of job time $A = \|a_{ij}\|$ the lower bound of a characteristic T is defined thus:

$$T(P^r) \geq \frac{\sum\limits_{j \in \tilde{Q}_r} \bigwedge\limits_{i=1}^{m} a_{ij}}{m} + \begin{vmatrix} t_1(R^r) \\ \cdots \\ t_r(R^r) \end{vmatrix}^{(r-m)+1}. \tag{24}$$

Here $\tilde{Q}_r$ is the set of all jobs from Q_r, $t_k(R^r)$ is the moment of the ending of the k-th job in the order of jobs from R^r, calculated according to section 5, $|\ldots|_s$ is a serial LD-column of a rank s.

14.

Some economic models of an industrial type have been considered in [1] – [5]. CL-models of several concrete classes of other systems are studied in [6] – [8]. The generalizing consideration is undertaken in [9].

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Nicephorus Blemmydes on the Holy Trinity and the Paraconsistent Notion of Numbers: A Logical Analysis of a Byzantine Approach to the *Filioque*

Basil Lourié

St. Petersburg, Russia

e-mail: hieromonk@gmail.com

Abstract:
The paper deals with the most controversial – in the modern scholarly discussion – episode within the Byzantine polemics on the *Filioque*, Nicephorus Blemmydes' acknowledgement of proceeding of the Spirit *through* the Son providing that the Son be considered as generated *through* the Spirit. The logical intuition behind this theological idea is explicated in the terms of paraconsistent logic and especially of a kind of paraconsistent numbers called by the author "pseudo-natural numbers". Such numbers could not be interpreted *via* the notion of ordered pair. Instead, they imply a known (first described by Emil Post in 1941) but still little studied logical connective ternary exclusive OR.
Keywords: Nicephorus Blemmydes, Filioque, Spirituque, Triadology, paraconsistent logic, paraconsistent numbers.

...cette intuition du nombre pur, la seule qui ne puisse nous tromper.

Henri Poincaré [42, p. 122]

1. Introduction: Symmetric Formula "Through Each Other"

The modern view that the dogma of the Trinity must be – in some way, at least, – logically consistent is not that of Byzantine Patristics. For Byzantine Fathers, it was normal to insist that the Trinity is beyond the human mind, because God is beyond any rational consistency. Thus, an "implicit inconsistence" of the Trinity in Plantinga's sense[1] was taken as granted. Nevertheless, the explicit logical frames of the dogma were often presented as consistent as possible.

As a rule, the Fathers have avoided usage of openly inconsistent logical categories and preferred to follow a kind of Niels Bohr's Correspondence Principle, that is, to use classical logical notions in a non-classical way [45], [3]. Such a standard has been established by Cappadocians in their anti-Eunomian polemics and was normally kept by Byzantine participants of anti-Latin polemics on the *Filioque*. The cases of an explicit discussion of inconsistency of the Cappadocian Triadology were, in Byzantine Patristics, relatively rare, even though sometimes quite important (especially those by Dionysius the Areopagite) [33].

The case of Nicephorus Blemmydes was somewhat different. In his *Filioque* polemics, he eventually reopened discussion on the very notions of "Son", "Spirit", and "Father", without limiting himself to discuss their mutual relations.

Blemmydes recalled the definitions of hypostasis through the notion of energy instead of the notion of essence (nature). Indeed, providing that the energy is "the movement of the essence"[2], such definitions must be mutually equivalent. The resulted "non-static" definitions of hypostasis (applied in particular to the hypostasis of the Spirit) – far from being invented by Blemmydes but taken verbatim from Athanasius of Alexandria[3] – were much more useful for the further discussion of the proceedings of the two hypostases (those of the Son and the Spirit) from the one (that of the Father).

In this way, Blemmydes elaborated a compromise understanding of the *Filioque*, which has, however, never been explicitly discussed in full (not fragmentary) by anybody, without exception of Blemmydes' followers, Gregory of Cyprus and the Palamites. The present article is aimed at such an explicit discussion of Blemmydes' central triadological concept, namely, its symmetrical formula of proceeding of the Son and the Spirit "through each other", or, more literally, "(any?)one (of the two) through another" – διὰ θατέρου θάτερον: "…si le Verbe et l'Esprit <existent> à partir du Pèrecomme du principe sans que l'un des deux <existe> par l'autre (μὴ διὰ θατέρου θάτερον), une division s'introduitdans la Divinité" (*Epistle to Theodore Laskaris*, 10 [48, v.1,pp. 346/347 txt/tr.]). The symmetry of this formula could be understood as either complete or partial, depending on our understanding of θάτερος as either "*any*one of the two" (the normal meaning unless otherwise is specified) or "a specific one of the two" (the meaning that could be specified with the context).

I have previously noticed that this formula is a unique one [57]. It could be interpreted in different ways, depending on the context. On the one hand, it could be put into a Latin context, as John Bekkos really did, but even the modern critics of Blemmydes agree that this was a too much forced interpretation of Blemmydes' original thought [25][4]. But, on the other hand, what about the Palamite reading of Blemmydes?[5] Was the Byzantine Orthodoxy read into Blemmydes' works by the Palamite readers – as now Jean-Claude Larchet claims – or did it actually exist in Blemmydes' theological thinking?

The most of considerations put forward so far *pro* and *contra* Blemmydes' orthodoxy are not more than a balance of probabilities. Anyway, Blemmydes normally does not decline from the already established Patristic language: even its "worst" (from, so-to-say, Larchet's viewpoint) formulations allegedly confusing the notions of hypostasis and energy are based on Athanasius. No Blemmydes' critic was able to take him at his words.

The value of Blemmydes' unique "symmetric formula" is therefore exceptional. It was both original and never quoted by either Gregory of Cyprus or later Blemmydes' followers. It is therefore the only "difficult" theological statement by Blemmydes.

There could be no doubt that it would have been not useful in any polemical context, where only familiar formulations were acceptable. This would explain why this formula has been put aside by Gregory of Cyprus and Gregory Palamas. Indeed, Blemmydes himself did not use it in polemics.

Be this as it may, Blemmydes' formula needs to be checked against its patristic background. If it turns out to be impossible with the usual methods of patrology – that is, tracing its prehistory in earlier theological texts – we have to explore the underlying logical model implied by Blemmydes.

Such a study will be inevitably limited in respect of Blemmydes' own "secret thoughts" (in what extent he realised himself the logic he followed, and so on). The latter, however, is not a matter of any scholarly interest – at least, in patrology. The *histoire des idées*has a logic of its own, and it does not matter in what extent any personality who contributed to it was psychologically fitting him – or herself with the intellectual flow of history. We have to study logic without a psychological commitment.

Fortunately, a recent publication of the previously unknown Blemmydes' texts by Michel Stavrou provides a decisive witness that the completely symmetrical understanding of the Blemmydes' formula ("through each other") was not that of Blemmydes himself. His original meaning was "a specific one of the two" (proceeding of the Spirit though the Son but not *vice versa*)[6]. Therefore, his own opinion on the mediatory role of the Son in the proceeding of the Spirit was either somewhat "subordinationist" (I would use this term as derived from the term "order", but Michel Stavrou prefers to say of a "réciprocité asymétrique" between the Son and the Spirit[7]) or still not completely clarified even to himself. Be this as it may, we know, from the further development of the Byzantine theology, that it was Blemmydes who gave an impetus to symmetrical explanations of the mutual relations between the Son and the Spirit.

2. The Basic Problem of Triadology: With or Without Pairs?

In Triadology, one counts to three but has a very big problem how to pass two. Can we speak about three without previously speaking about two? The "symmetric formula" would imply an answer like "Yes, but, in this case, the second will be also the third, and the third will be also the second".

The problem of order within the Trinity (especially who is there the second and who is the third) has been dealt with at length in *ca* 1335 by Gregory Palamas. He insisted (and argued with the testimonies taken from the Cappadocian Fathers and Ps.-Chrysostom = Severian of Gabala) that the Latin *Filioque* implies an order within the divine nature, whereas no order between the three hypostases is allowable in this sense (that is, in the sense of ἐκ τῆς φυσικῆς ἀκολουθίας / "from the natural consequence"). He avoided here, nevertheless, the explicitly paradoxical symmetric formula with its "quantum superposition" of the two proceedings[8]. In general, Palamas argues here "from the Fathers" but not from any logical system. This could be sufficient for demonstrating that, in the Cappadocian Triadology, any "natural" order between the Son and the Spirit is forbidden, but not for explaining why.

In *Filioque* doctrines, however, the intermediary step of counting to two has never been skipped, neither in the old Carolingian two-principle *Filioque*, nor in the *tanquam ab uno principio* of the 1274 Council of Lyon. Both kinds of *Filioque* were perfectly consistent, from a logical point of view. There were, however, some equally consistent alternatives to the *Filioque* Triadology. Let us briefly consider them all starting from the Triadologies implying pairs.

Both early Latin *Filioque* with its two "principles/beginnings" within the Holy Trinity (the Father and the Son are two different "principles" of the Holy Spirit: thus in the *Libri Carolini* (s. on them [18]) and the mainstream Latin doctrine before the 1274 Lyon Council, including the lifetime of Blemmydes) and Council of Lyon's 1274 doctrine of *tanquam ab uno principio* (the Father and the Son form a unique "principle" of the Spirit) imply some pairing. In the first case, these are the pairs of the Father and the Spirit and the Son and the Spirit = (F + Sp) + (S + Sp) = (1 + 1) + (1 + 1). In the latter case, this is the pair of the Father and the Son followed by the pair of them both and the Spirit = (F + S) + Sp = (1 + 1) + 1.

Given that the μοναρχία of the Father is out of question (that is, the Father must remain on the first place everywhere), one can easily see that only the two following variants of pairing are left: (F + Sp) + S = (1 + 1) + 1, — and (F + S) + (F + Sp) = (1 + 1) + (1 + 1).

In sum, there are logically possible only four "pairing" Triadologies which correspond to two schemes of pairing (Table 1). We have just discussed two Triadological doctrines where the schemes of pairing were different.

Table 1.

Doctrine	Scheme of Pairing	Historical Realisation
(F + Sp) + (S + Sp)	(1 + 1) + (1 + 1)	Earliest *Filioque* doctrine
(F + S) + (F + Sp)		Opposition to Gregory of Cyprus; Meletius Pegas *et al*.
(F + S) + Sp	(1 + 1) + 1	Lyon's *Filioque* doctrine (*tanquam ab uno principio*)
(F + Sp) + S		Ethiopian doctrine *Qəbat*

The two remaining Triadological doctrines implying pairing are the following.

The doctrine (F + Sp) + S, that is, "the Son is born from the Father through the Spirit", is another version of *tanquam ab uno principio*, where this "principle" is, however, the Spirit and not the Son. It has been realised in the Ethiopian seventeenth-nineteenth-century theological doctrine *Qəbat* ("Unction": the Son is born through the function of the Spirit)[9]. It has never been in consideration in the middle and late Byzantium or among the Latins. In the eyes of the Ethiopian adherents of this doctrine, it was not without support in the works of Cyril of Alexandria and Cappadocian Fathers, and we will see that such claims were not completely unfounded. Nevertheless, such an attitude was hardly possible anywhere in Byzantium during the whole period of the *Filioque* polemics.

The doctrine (F + S) + (F + Sp), that is, the generation of the Son and the procession of the Spirit are absolutely independent from each other, appears shortly after Blemmydes, within the Byzantine opposition to the Triadology of Gregory of Cyprus and his 1285 Blachernae Synod[10]. The imminent victory of Palamism led to a temporary suppression of this doctrine, but it will reappear near 1600 as a result of the Triadological quarrel between Maximus Margounios and Gabriel Severus supported by Meletius Pegas[11]. The latter will become the main responsible for its *de facto* canonisation in the nominally Orthodox textbooks until the "rediscovery" of Gregory of Cyprus in the twentieth century. In Pegas and textbooks that followed him, the topic of uncreated energies became completely absent (whereas already Maxim Margounios made the first step in this direction limiting the divine energies to the revelation to the creation without allowing them to be *in divinis*).

This doctrine becomes vulnerable to the same arguments that were used against the Latin Carolingian *Filioque* doctrine, namely, that a distinction between the Father and the Son as the two "principles" would imply a division within the Trinity. Here, in a similar way, a division between the Son and the Spirit as the two separate "products" of the Father would imply an analogous division.

In fact, now, we have exhausted the list of the consistent and paradox-free treatments of the mutual relations between the hypostases within the Holy Trinity. Any other paradox-free approach would lead to either explicit Arianism or explicit Sabellianism. The traditional Byzantine approach was, however, still different.

3. Why Cappadocian TriadologyBecame Incomprehensible

The Byzantines reached the discussions on the *Filioque* when the logical and philosophical language of their theology was not in its best shape. We have to mention this, even though there is no room here to go into details, not to say that the most of these details are still waiting for being studied.

The key to the original explanation of the unity in the trinity given by Cappadocian Fathers was definitively lost already in the sixth century, when – especially under the high pressure of John Philoponus' influence [58], [21] – it became habitual to treat the unique essence/nature of the godhead as a common ("second") nature in the Aristotelean sense. There was, then, a range of interpretations of what this "Aristotelean sense" means, within the span between, so-to-say, "nominalism" and "realism", – but all of them were reducing the natural unity between the divine hypostases to the same level as the natural unity between three men, whatever explanation of the latter kind of unity would have been provided [53]. This was certainly not enough for explaining the unity of the Holy Trinity (unless one would have wished to adopt Philoponus' own doctrine of the "Tritheism").

Therefore, since the seventh century as the latest, Chalcedonian theologians were trying to elaborate other conceptions, as it was first witnessed by John of Damascus ([6], [49], [10], but s. criticisms of [7] in [23]). We have now to skip this part of the story but need to notice that the very intervention of Nicetas Stethatos into the 1054 discussion with Cardinal Humbert turned out to be an exacerbation of these Byzantine problems with the logical inconsistence of their Triadology[12]. Probably, Dirk Krausmüller is not always right claiming the "nominalist" trend in the understanding of the divine nature "heterodoxy" (because there is no bijective correspondence between the philosophical/logical and theological concepts) but, at least, usage of such categories has made the logical construct of Triadology to be more and more far from Cappadocian Fathers.

Nicephorus Blemmydes avoided reopening the whole issue, but in his controversial formula, he puts his finger on its main logical knot, namely, the problem of logical consistency.

4. The Two *Spirituque*

In the realities of the twentieth century, the slogan of symmetric Triadologies became *Spirituque* "and from the Spirit" by Paul Evdokimov[13] – patterned, of course, after *Filioque*. Such a label is somewhat misleading, because it does literally mention a diametric opposition to the *Filioque*, which is the doctrine *Qəbat*. The *Qəbat* and *Filioque* doctrines mirror each other, whereas the *Spirituque* doctrines are always symmetric, presuming *both* proceeding of the Spirit though the Son and begetting of the Son through the Spirit.

It would be difficult and irrelevant to our purpose trying to figure out Evdokimov's original meaning of the *Spirituque*. Anyway, his uncritical relying on Bolotov makes *ipso facto* Larchet's criticisms[14], at least, partially justified. Bolotov was not only "anti-paraconsistent" but rather "positivistic". In a consistent framework, any symmetric approach to two "second" hypostases would have been acceptable with a price of reducing their self-standing reality, that is, with a concession to the Sabellianism. It is worth noting that the *Filioque* doctrine *tanquam ab uno principio* has sounded Sabellianic to Byzantine Fathers [34]. No wonder that the *Spirituque* approach acquired a support from some Catholic theologians including such famous figures as Yves Congar (after some hesitations: s. [16, p. 79]) and especially Leonardo Boff who produced a detailed triadological doctrine in the *Spirituque* line supported with both patristic and modern witnesses [2, pp. 13, 106-108, 180-182, 224-227, 249-253, 286].

Nevertheless, the question is not as simple as that. Alexander Golitzin published, to my opinion, the most important contribution shedding light on the real dimension of the problem. Golitzin was commenting on a unique passage of Gregory of Nazianzus where the Holy Trinity is compared to the first human family in the way that Eve becomes an image of the Spirit who gives birth to the Son[15]. This passage, according to Golitzin, is to be read within a large mystical Semitic/Syriac tradition where the Holy Spirit is "Mother"[16] and also within the mainstream Christian liturgical tradition (referring itself to the Annunciation narrative in Luke), where the flesh of Christ appears after an invocation of the Holy Spirit [15].

If we imagine Alexander Golitzinbeing an Ethiopian theologian belonging to the *Qəbat* faction, he could provide an ample patristic dossier favouring the generation of the Son through the Spirit. The real Golitzin puts forward, instead, a more balanced view endorsing Leonardo Boff's *Spirituque*. What is especially helpful, he recalled Dumitru Stăniloae's already elaborated exposition of the "non-causal reciprocity" between the Son and the Spirit[17], based, in turn, on Joseph Bryennios' (*ca* 1350–1431) Trinitarian theology[18].

Stăniloae, following Joseph Bryennios, distinguishes between two kinds of interpersonal relation within the Trinity, causal (between the Father and the Son and between the Father and the Spirit) and non-causal (between the Son and the Spirit). It is obvious that the reciprocity between the Son and the Spirit meant in the symmetric formula is non-causal, either: it implies that the Father is the only "cause" in the Trinity.

Stăniloae quotes from a long Bryennios' exposition: "The Son, because he is the one who is the Son, alone possesses the name of Son *vis-à-vis* the Father, for he is the Son of the Father only, not of two; but the name of Word which belongs to the Son alone within the Holy Trinity has reference not only to the Father as the one who is Mind, but also to the Spirit in another way…"[19]. And, in the same manner, the Spirit is He Who Proceeds only *vis-à-vis* the Father who caused him to proceed, whereas being the Spirit of both Father and Son[20].

The elaborated Triadology by Joseph Bryennios – not only in the *Hortatory Sermon* but also in some other works, including the *Twenty One Sermons on the Holy Trinity* – provides, to my opinion, an appropriate context for a deeper understanding of post-Blemmydian symmetric Triadology. Bryennios discusses at length the non-causal relations and the (in)existence of the physical order within the Holy Trinity.

Stăniloae and, after him and following him, Golitzin, provided us with the optics having a sufficient resolution for perceiving the inner traditionalism behind Blemmydes' apparent innovation, that is, his symmetric formula.

Nevertheless, the question remains: what is the logic implied in such a symmetric Triadology? It is already clear from the above that it is not any logic avoiding the contradictions, but now we need to define the kind of paraconsistent logic we are dealing with.

5. From a Logical Point of View

5.1. "Set-theoretical" Reformulation of the Problem

The symmetric formula implies a severe logical difficulty — at least, from a "classical" logical point of view. It does not allow counting to three and, therefore, makes the number three in "Trinity" unintelligible. These are not bad news, judging from a Cappadocian perspective, but certainly not easily digestible for Blemmydes' contemporaries.

The problem is the following. To be able counting to three, we have, normally, to count to two and, then, to repeat the same procedure when reaching three. In the modern set-theoretical language, we can say that, to be able to count, we need oriented pairs, that is, elementary sets having two elements, where one element is chosen to be the first and the remaining element is, thus, the second. Without this, no natural row of numbers is possible, neither any row of numbers known to our modern mathematics.

In the "symmetric" counting, no oriented pair and no pair at all are possible. The elementary set contains here three elements, not two. One of these three elements is chosen to be the first (the Father)[21], whereas no element is chosen to be the second or the third. Alternatively, one can say that two elements simultaneously are the second ones.

Evidently, we have left the ground of classical logic in general and any usual mathematical logic in particular. We have, nevertheless, to make explicit the logic of the symmetric formula as it is.

Our explanation will go through the following steps:

1. Explanation of the logical connective implied: what means "to choose exactly one (Father = "cause") from three", that is, what is the difference between this logical procedure and the choice of exactly one from two, in a three-element set, repeated two times.
2. What kind of sets and, therefore, what kind of numbers we are dealing with.
3. What kind of non-classical logic is implied.

5.2. The Principle of "Sabellianism": Any Pairing Scheme in Triadology Implies an Implicit "Sabellianism"

There are three different kinds of logical connective OR (disjunction) but only two of them are widely known: the inclusive disjunction ("at least, one of the two") and the ordinary (that is, binary) exclusive disjunction ("exactly one of the two"). The third kind of disjunction has been first described – in the modern logic – by Emil Post in 1941 did not become widely known – at least, among the logicians – until recently [39]. This is the ternary exclusive disjunction ("exactly one of the three") and its generalisation up to *n*-arity ("exactly one of the *n*").

The two kinds of exclusive disjunction show drastically different behaviour (truth-functions) starting from the arity 3 and, then, at the each odd value of the arity. It is almost self-evident that the mutual relations of the hypostases of the Trinity are to be described with the ternary exclusive OR and not with any other disjunction: a given hypostasis is exactly one of the three. Nevertheless, let us demonstrate it in a more detailed way. Let us begin with a demonstration of a principle concerning the possible triadological meaning of the ordinary (binary) exclusive disjunction.

At the arity 2, the two kinds of exclusive disjunction are not distinguishable and have the same truth-function that is described in Table 2. Let us consider, as an example, two arbitrary hypostases of the Trinity, say, the Father and the Son. Let the proposition φ_1 will be "the hypostasis *X* is the Father" and the proposition φ_2 will be "the hypostasis *X* is the Son". These two propositions cannot be true simultaneously. Thus, the truth-function is the following (T means "true", F means "false", and $\oplus$ is the symbol of the ordinary exclusive disjunction):

Table 2.

φ_1	φ_2	$\varphi_1 \oplus \varphi_2$
T	T	F
T	F	T
F	T	T
F	F	F

In our example, we have had a choice between the two possibilities only: the hypostasis *X* could be either Father or Son or, alternatively, neither of them, but there was no third value for the notion "hypostasis". The exclusive binary disjunction allows only two situations to be true: when either φ_1 or φ_2 is true, whereas the remaining proposition is false. The situation where both φ_1 and φ_2 are true is false, that is, not allowed.

So far so good. Let us consider a more complex situation that corresponds to that of the Trinity. Thus, let us include the proposition φ_3 "the hypostasis *X* is the Spirit". The ordinary exclusive disjunction would presuppose the choice between the pairs. The order of these pairs does not matter. Let us consider the sequence $(\varphi_1 \oplus \varphi_2) \oplus \varphi_3$; s. Table 3.

Table 3.

φ_1	φ_2	φ_3	$(\varphi_1 \oplus \varphi_2) \oplus \varphi_3$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

As in the previous example, the hypostasis *X* could not take more than one value: it is the Father or the Son or the Spirit, but never any two of them nor all the three. Our connective, however, whereas not allowing any two-value situation for *X*, does allow the three-value one: the first row of Table 3 contains the value of the truth-value function that renders the ordinary exclusive disjunction inapplicable to the Trinity.

The ordinary (binary) exclusive disjunction makes choice between the pairs. If there are more than two objects to choose between them, it repeats the choice from a pair. Thus, at an arity more than 2, this kind of exclusive disjunction is nothing but a reiteration of the binary disjunction. This is why the choice from three is performed in two binary steps, where the second step is a choice between the result of the former step and the remaining third object. This is why the first row of Table 3 contains such a value of the truth-function. At the first step, when there were two propositions claimed to be true, our connective results in the truth-value "false", but then, at the second step, it has to deal with this resulting from the former choice false proposition and the apparently true third proposition, that results in the decisive truth-value "true". This result is to be repeated at all odd numbers of options. If the number of options is even, the choice of all options simultaneously is impossible.

If in the Trinity there existed the pairs, our hypostasis *X* could be simultaneously the Father, the Son, and the Spirit. Let us notice that we have just demonstrated logically (using the *reductio ad absurdum* method) the following theological principle:

The Principle of "Sabellianism"

- Any triadological doctrine on the "triune God" (that is, any non-Arian Triadology) implying any kind of pairing between the hypostases would be intrinsically "Sabellianic" (to the Byzantine taste[22]).

5.3. A Logical Connective Disallowing the Pairs: Ternary Exclusive OR

If we correct Table 3 making a unique change, namely, changing the truth-value in the first row from T to F, we obtain another truth-function that corresponds to another kind of disjunction, the ternary exclusive OR, the connective that never allows to choose all the three from three.

The ternary exclusive OR forbids the choice of all options simultaneously even at the odd numbers of options, which would be impossible with reiteration of the choice between pairs.

The three hypostases of the Trinity are connected with the ternary exclusive OR and not with the ordinary (binary) exclusive disjunction. This means that there are no pairs in the Holy Trinity.

Instead of ordered pairs, the ternary (*n*-ary) exclusive OR creates the groups "the chosen one + all others", where these "all others" are different from each other but not distinguishable, except the unique respect: they are distinguishable only in contrast with the unique element of the set that is chosen to be first, but all of them are equally “second”.

Such an apparently little change in the table of truth-values makes the very idea of such numbers as “one”, “two”, and “three” drastically different from the idea of natural numbers. Indeed, we have been foretold by the Cappadocians and Evagrius that the numbers in the Trinity are not the same numbers as those in the world. Without going any deeper into the Triadology of the Cappadocian Fathers, I will provide one quote from Evagrius, where the Cappadocian “mathematical” ideas are formulated in the most explicit way[23]:

VI, 11. La triade numérique est accompagnée d’une tétrade, mais la Trinité sainte n’est pas accompagnée d’une tétrade ; elle n’est donc pas une triade numérique. 12. La triade numérique est précédée d’une dyade, mais la Trinité sainte n’est pas précédée d’une dyade ; elle n’est pas, en effet, une triade numérique. 13. La triade numérique est constituée par addition d’unités sans substance ; mais la Trinité bienheureuse, ce n’est pas par addition de telles unités qu’elle est constituée ; elle n’est donc pas une triade qui soit avec nombres.	ܝܐ — ܠܬܠܝܬܝܘܬܐ ܕܡܢܝܢܐ ܠܘܝܐ ܐܪܒܥܝܘܬܐ. ܠܬܠܝܬܝܘܬܐ ܕܝܢ ܩܕܝܫܬܐ ܠܐ ܠܘܝܐ ܐܪܒܥܝܘܬܐ. ܠܝܬܝܗ ܡܟܝܠ ܬܠܝܬܝܘܬܐ ܕܡܢܝܢܐ܀ ܝܒ — ܠܬܠܝܬܝܘܬܐ ܕܡܢܝܢܐ܆ ܩܕܡܐ ܬܪܝܢܝܘܬܐ. ܠܬܠܝܬܝܘܬܐ ܕܝܢ ܩܕܝܫܬܐ܆ ܠܐ. ܩܕܡܐ ܬܪܝܢܝܘܬܐ. ܠܝܬܝܗ ܓܝܪ ܬܠܝܬܝܘܬܐ ܕܡܢܝܢܐ܀ ܝܓ — ܠܬܠܝܬܝܘܬܐ ܕܡܢܝܢܐ܆ ܒܪܘܟܒܐ ܕܚܕ ܚܕ ܕܠܐ ܩܢܘܡ ܡܬܩܢܝܐ. ܬܠܝܬܝܘܬܐ ܕܝܢ ܛܘܒܢܝܬܐ܆ ܠܘ ܒܪܘܟܒܐ ܕܝܚܝܕ̈ܝܐ ܕܐܝܟ ܗܠܝܢ ܡܬܩܢܝܐ. ܠܝܬܝܗ ܡܟܝܠ ܬܠܝܬܝܘܬܐ ܕܡܢܝܢܐ܀

This quotation makes explicit the basic difference between the numbers meant in the Cappadocian Triadology and the natural numbers: unlike the latter ones, the former ones are not ordered. The “three” in the Holy Trinity does not imply that there is a “two” before it nor a “four” after.

Such features do not allow an idea of ordinality in any known mathematical meaning. This, in turn, follows that they do not allow an idea of cardinality (in any already known mathematical meaning), either (because there is no definition of cardinality without a reference to the notion of ordinality). These features are quite understandable due to the properties of the ternary exclusive OR. Nevertheless, we have to make explicit the “theory of numbers” implied.

5.4. “Pseudo-natural Numbers” and Their Set-theoretic Interpretation

I will call the numbers implied in the Cappadocian Triadology “pseudo-natural” due to their deliberate similarity with the natural numbers. To my knowledge, there are no similar objects in the modern mathematics (cf. [36], [44]), and therefore, we have to interpret them in a usual set-theoretic way. We will begin with van Neumann’s definition/interpretation[24] of the natural numbers *via* the ordered pairs, because such an approach would be especially useful for demonstrating the consequences of suppressing the ordered pairs from numerology.

According to Casimir Kuratowski’s definition [24] (cf., for a larger historical context, [19, pp. 23-26]), the ordered pair (a, b) (where a is the first element and b is the second) is the set $\{\{a\},\{a, b\}\}$, where $\{a, b\}$ is the unordered set (pair) formed with the same elements.

Johann von Neumann proposed the following definition of the natural numbers based on the notion of ordered pair (although still without knowing Kuratowski’s 1921 paper[25]):

- The number 0 is defined as the empty set { },
- The successor function is defined as $S(a) = a \cup \{a\}$ for every set a,
- Each natural number is equal to the set of all natural numbers less than it:

$0 = \{\ \}$,
$1 = 0 \cup \{0\} = \{0\} = \{\{\ \}\}$,
$2 = 1 \cup \{1\} = \{0, 1\} = \{\{\ \}, \{\{\ \}\}\}$,
$3 = 2 \cup \{2\} = \{0, 1, 2\} = \{\{\ \}, \{\{\ \}\}, \{\{\ \}, \{\{\ \}\}\}\}$,
$n = n{-}1 \cup \{n{-}1\} = \{0, 1, \ldots, n{-}1\} = \{\{\ \}, \{\{\ \}\}, \ldots, \{\{\ \}, \{\{\ \}\}, \ldots\}\}$, etc.

It is clear from the above that, for the row of natural numbers, the existence of ordered pairs is a *conditio sine qua non*.

In our "pseudo-natural" numbers, we have, instead, "pseudo-ordered" pairs, which are the pairs where only the first element is defined, whereas all other elements of the set are the second. This is a paraconsistent conjunction based on the contrary (not contradictory) opposition: from a classical point of view, it is impossible that there are more than one "second" elements, whereas there is no problem, if none of given elements is the second one.

Thus, the pseudo-ordered pair is to be defined as following. In a set of *n* elements, there is one element chosen to be the first, *a*; the remaining elements (designed with the letter *b* with an appropriate index) are in amount of *n*–1. Thus, the pseudo-ordered pair is

$$\left(a, \bigwedge_{n-1} b_{n-1}\right) = \bigwedge_{n-1} \{\{a\}, \{a, b_{n-1}\}\}$$

The above formula is paraconsistent: it does not design *n*–1 pairs, but only a unique pair with *n*–1 "second" elements.

For the case of the Trinity, *n* = 3.

It is clear that the pseudo-natural numbers do not form any row. Instead, their set has only one "ordered" component: the element chosen to be the first. All other elements of their set are equally the "second".

Thus, as Evagrius pointed out, there could be neither "two" nor "four" flanking the "three" in the Holy Trinity.

Let us notice that these paraconsistent relations in the Holy Trinity are not causal. In their respective causal relations, both Son and Spirit are completely distinct without forming any paraconsistent relations. However, this consistent and "classical" reasoning in Triadology is placed within a non-classical concept (our pseudo-natural numbers), exactly according to Niels Bohr's Correspondence Principle.

However, in non-causal relations, the Father is not necessarily the first in the Holy Trinity. Many Byzantine authors, whereas not Blemmydes, dedicated detailed explanations to why there is no "physical order" among the hypostases of the Holy Trinity, that is, why any hypostasis could be counted as the first one[26]. Thus, theoretically, there is not only one choice of the first element (discussed by Blemmydes) but all the three, and the resulting number of the pseudo-ordered pairs in the paraconsistent conjunction is equal to the number of permutations (ordered combinations) of two elements from *n*,

$$P_n^2 = \frac{n!}{(n-2)!}$$

In the Holy Trinity, where *n* = 3, this results in 6. If one element from three is already chosen, we have to replace *n* in the above formula with *n*–1, which results in 2: the two paraconsistent non-causal conjunctions covered by the symmetric formula.

6. Conclusion

The Byzantine patristic tradition is certainly rich enough to provide us with an elaborated theory of pseudo-natural numbers. Blemmydes, however, was not a theoretician of it. His merit consists in making some first steps in its direction when it became semi-forgotten by his contemporaries.

There is no room here to go deeper into analysis of both theological, set-theoretic, and logical problems related to the pseudo-natural numbers[27]. The purpose of the above study was to make visible a powerful flow of patristic logical thought to whom Blemmydes demonstrated an imperfect but unusual, for his epoch, sensitivity.

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Notes

1. Cf. Plantinga's distinction between three kinds of contradiction in theology (explicit, formal, and implicit) in [40, p. 12-24] (first published in 1974).
2. The earliest witness of this definition is the textbook *Doctrina Patrum de incarnation Verbi* (*ca* 700) where it is ascribed to Gregory of Nyssa (now unknown in his preserved works): Ἐνέργειά ἐστι ποιά τις κίνησις τῆς οὐσίας. <…> Ἐνέργειά ἐστι κίνησις δραστική. κίνησις δέ ἐστι παράλλαξις τοῦ προτέρου ["The energy is as if some movement of the essence. <…> The energy is an active movement, whereas the movement is an alteration of the former" (*sc.*, state)] [9, p. 258, lines 4, 6-7].

3. See the most detailed analysis of the relevant passages of Blemmydes in [35]. The key Athanasius' passage (*Epistula I ad Serapionem*, 20–21)was here referred to by Makarov according to *PG* 26, 577C-580A, still without an access to the recent critical edition, which made clear that a striking difference between Athanasius original texts and its quotation in both Blemmydes and Gregory of Cyprus is not an "alteration" made by Blemmydes [35, pp. 206-207] but a different variant reading in manuscripts of Athanasius (where the Holy Spirit either "is the living energy" or "…being the life, that is the energy…"): [πνεῦμα] ἑνὸς γὰρ ὄντος τοῦ ζῶντος λόγου μίαν εἶναι δεῖ τελείαν καὶ πλήρη τὴν ἁγιαστικὴν καὶ φωτιστικὴν ζῶσαν [*variant reading* ζωὴν οὖσαν] ἐνέργειαν αὐτοῦ καὶ δωρεάν, ἥτις ἐκ πατρὸς λέγεται ἐκπορεύεσθαι... (ch. 20:4; [46, pp. 29-31]).
4. Cf. my answer to this and other criticisms by Larchet [30, pp. 502-506], as well as my review of tome I of [48]: [31, esp. p. 426], where I endorsed Stavrou's claim that Blemmydes' theological work was not an "…evolution mais approfondissement ou *involution* de la pneumatologie byzantine" (quoted [48, v. 1, p. 117]).
5. Gregory Palamas' explicit quotation from Nicephorus Blemmydes has been misinterpreted by John Meyendorff and other scholars but is eventually rightly identified by Ioannis D. Polemis [43].
6. A series of syllogisms without title entitled by the editor "Autres syllogismes sur la procession du saint Esprit" and dated to the same period as Blemmydes' two major treatises on the procession of the Holy Spirit (1255–1256). Here, in the syllogism 4, Blemmydes explicitly denies the symmetry: Εἰ μὴ διὰ τοῦ Υἱοῦ τὸ Πνεῦμα τὸ ἅγιον, ἔσται διὰ τοῦ Πνεύματος ὁ Υἱός·ἀλλὰ μὴν τοῦτο οὐκ ἔστι, τὸ ἕτερον ἄρα ("Si l'Esprit saint n'existe pas par le Fils, le Fils existera par l'Esprit ; mais il n'est pas ainsi, donc l'autre hypothèse [estvraie]") [48, v.2, p. 224/225 txt/tr.]. Moreover, in the syllogism 11, he uses almost the same formula as διὰ θατέρου θάτερον — ἑτέρας διὰ τῆς ἑτέρας, — but in a perfectly transparent context: Εἰ μὴ διὰ τοῦ Υἱοῦ τὸ Πνεῦμα τὸ ἅγιον, τόπῳ περιγραφήσεται ἡ Τριάς· αἱ γὰρ ὑποστάσεις τοῦ Υἱοῦ καὶ τοῦ Πνεύματος, μὴ ἑτέρας διὰ τῆς ἑτέρας προϊούσης, τόπον τινὰ οὐ περιλήψονται ἐν ᾧ θατέρα ἐσεῖται, ἀλλὰ παρὰ μέρος ἀντιλήψονται τούτου ("Si l'Esprit saint n'existe pas par le Fils, la Trinité sera localement circonscrite. En effet, les hypostases du Fils et de l'Esprit — aucune des deux ne venant [à l'être] par l'autre — n'embrasseront pas un lieu dans lequel existera l'une des deux, mais c'est en partie qu'elles s'attacheront à ce lieu") [48, v. 2, pp. 226/227 txt/tr.]. I am very grateful to Michel Stavrou for pointing me out these formulations by Blemmydes.
7. As he suggested in an e-mail to me on 21 June 2016.
8. Gregory Palamas, *First Apodictic Sermon on the Procession of the Holy Spirit*, 32-33; ed. by B. Bobrinsky in [54, pp. 60-64] (quotation pp. 61, line 24).
9. See, as a short introduction, [50]. As the best introduction to the theological doctrine, s. [13].
10. For historical overview, [38], [26]; cf. [30].In [30] I put forward a guess that the leading theologian of this group was Georges Moschampar. Unfortunately, there is no, so far, any detailed study of the theology of the adversaries of Gregory of Cyprus and its possible relation to the Triadology of Akindynos (a direct adversary of Gregory Palamas, whose pseudo-Photian quotations show a similar or the same theology).
11. No detailed study so far. Cf. some brief observations in [29, pp. 183-184]. Recently, Marcus Plested recalled this discussion but without any analysis [41, pp. 146-147].
12. Cf. now a series of Dirk Krausmüller's articles on Nicetas Stethatos (the third one is still in preparation).
13. "Le Père engendre le Fils avec la participation de l'Esprit Saint et il spire l'Esprit Saint avec la participation du Fils" [11, p. 72]; "…le Fils est la condition trinitaire de la spiration du Saint-Esprit par le Père, l'Esprit Saint est la condition trinitaire de l'engendrement du Fils par le Père. L'innascibilité, la génération et la procession sont sans confusion ni séparation un seul acte tri-un de Révélation, avec la participation simultanée et réciproque des Trois" [11, p. 75]. Evdokimov refers, here and elsewhere, to Vassily Vassiljevich Bolotov as his principal predecessor in this approach. See: [51], important translations and commentaries: [4], [55], [56].
14. "Il est inutile de dire que cette théorie, en décalage complet par rapport à la Tradition orthodoxe, est une innovation irrecevable qui ne fait qu'ajouter une erreur à une autre et des nouvelles confusions aux confusions précédentes" [27, p. 25-26] (quotation p. 26, n. 54). Larchet, however, equates this Evdokimov's and Bolotov's attitude with that of Blemmydes and Stavrou.
15. Gregory of Nazianzus, *Oratio* 31, *On the Holy Spirit* (*Theological* 5), 11: "What was Adam? A creature of God. What then was Eve? A fragment (τμῆμα) of the creature. And what was Seth? The begotten of both (Ἀμφοτέρων γέννημα). Does it then seem to you that Creature (πλάσμα) and Fragment (τμῆμα) and Begotten (γέννημα) are the same thing? Of course, it does not. But were not these persons consubstantial (ὁμοούσια)? Of course they were. Well then, here it is an acknowledged fact that those that are differently hypostasised may have the same substance (τὰ διαφόρως ὑποστάντα τῆς αὐτῆς εἶναι οὐσίας ἐνδέχεσθαι). I say this, not that I would attribute creation or fraction or any property of body to the Godhead (let none of your contenders for a word be down upon me again), but that I may contemplate in these, as on a stage, things which are objects of thought alone. For it is not possible to trace out any image exactly to the whole extent of the truth. But, they say, what is the meaning of all this? For is not the one an offspring, and the other a something else of the One (Οὐ γὰρ τοῦ ἑνὸς τὸ μὲν γέννημα, τὸ δὲ ἄλλο τι. Τίοὖν ;)? Did not both Eve and Seth come from the one Adam (οὐχὶ τοῦ αὐτοῦ Ἀδάμ)? Indeed, from whom else? And were they both begotten by him? No; but the one was a fragment (τμῆμα) of him, and the other was begotten (γέννημα) by him. And yet the two were one and the same thing; both were human beings; no one will deny that. Will you then give up your contention against the Spirit, that He must be either altogether begotten, or else cannot

be consubstantial, or be God; and admit from human examples the possibility of our position? I think it will be well for you, unless you are determined to be very quarrelsome, and to fight against what is proved to demonstration." Text: [12, pp. 294, 296], tr. [5].

16. Interesting, Boris Bobrinskoy whom Golitzin quotes among the like-minded theologians, was afraid of giving a pretext for speculations on the Spirit as "Mother": "Comme l'écrit Paul Evokimoff, l'Esprit n'est pas étranger au mystère de la relation Père-Fils, car celui-ci estune relation triadique et non dyadique [11, p. 77]. Il est essential de poser cela ainsi au début même de toute réflexion sur le mystère trinitaire, tout en veillant à ne pas tomber dans la spéculation gnostique d'une « maternité » divine et éternelle de l'Esprit" [1, pp. 271-272].
17. In a paper published in Romanian in 1970 and reprinted as ch. III "The Holy Trinity: Structure of Supreme Love" of [47, pp. 73-108, 231-234, esp. 105-106]. Stăniloae refused to see, in Blemmydes, the germ of the same reasoning as in Bryennios (as Stăniloae wrote in a personal letter to me *ca* 1989).
18. Stăniloae referred to Bryennios in a rather confusing manner, only to the Romanian translation by Metropolitan Grigorie Dascalul, where the work referred to is added as the 22th sermon to Bryennios' 21 sermons *On the Holy Trinity* [8, p. 346]. In fact, the quoted Bryennios' work is *Λόγος συμβουλευτικὸς περὶ τῆς ἑνώσεως τῶν ἐκκλησιῶν* (*Hortatory Sermon on the Union of the Churches*, 1422) [52, pp. 469-500]; dated by Ph. Meyer, *pace* N. Tomadakes; cf. [28, p. 30, n. 1].
19. …ὁ Υἱὸς, τὸ μὲν Υἱὸς ὄνομα τοῦτο, καθὸ Υἱὸς, πρὸς μόνον κέκτηται τὸν Πατέρα· Πατρὸς γὰρ ἐστὶν ἑνὸς Υἱοῦ μόνου, καὶ οὐ δυοῖν· τὸ δὲ Λόγος ὄνομα, ὃ μόνος ἐν τῇ ὑπερθέῳ Τριάδι πλουτεῖ, οὐ μόνον ἐστὶ τοῦ Πατρὸς καθὸ νοῦ, ἀλλὰ δὲ καὶ τοῦ Πνεύματος, καθ'ἕτερον τρόπον...
20. Joseph Bryennios, *Hortatory Sermon…*, [52, esp. 487-499 and chart 3 (σχῆμα Γ΄) at the end of the volume], quoted p. 487; tr. from [47, p. 105].
21. No Byzantine patristic author would agree even with the claim that the Father is the first, because, in the Holy Trinity, there is no "the first", "the second", and "the third" in the natural order. However, in our present purely logical — or rather "set-theoretical" — language we are daring to say that the Father is the first (in a "non-Byzantine" sense of the word), where "the first" means "the cause (αἰτία)".
22. That is, "Sabellianism" would be considered, by the Byzantine polemists, as an appropriate heresiological label for the corresponding doctrine. Of course, the charges of implicit "Sabellianism" were compatible with those of "Arianism", because any kind of pairing within the Trinity would lead to deepening of the opposition between this pair and the remaining hypostasis. However, such an "Arianism" has appeared as a secondary phenomenon.
23. Quoted according to the original recension S_2, but the text of S_1 does not contain any important differences: [17, pp. 221, 223]; s., for S_1, [17, pp. 220, 222].
24. Whether it is a definition or an interpretation depends on one's attitude towards the foundations of mathematics. To von Neumann himself as well as other set-theoreticians of the early 20th cent., it was a definition, to Poincaré it would seem an interpretation. For such objects as our pseudo-natural numbers Poincaré's argumentation against Couturat, Zermelo, and Russell is valid. Cf. [14].
25. I quote the standard modern formulation of von Neumann's definition. For the original one: [37].
26. See above references to Gregory Palamas (who quotes classical passages of Severian of Gabala on the topic) and Joseph Bryennios. For a more general introduction to the pseudo-natural numbers, see now: [32].
27. I would like only to enumerate here the most striking among them. In theology: provenance of the "caused" hypostases (the Son and the Spirit) in relation to the divine energies. In set-theory and logic: foundation axiom, countability, axiom of choice.

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Orthodoxy and Philosophy

Travis Dumsday received his doctorate in philosophy in 2010 from the University of Calgary. After a postdoctoral fellowship at the University of North Carolina Chapel Hill, he taught for a year at Livingstone College and then in 2012 took up a post at Concordia University of Edmonton. A convert to Orthodoxy from Presbyterianism (chrismated in 2009), Dumsday works in multiple areas of philosophy, including philosophy of science, metaphysics, philosophy of religion, and bioethics.

Tudor Petcu: With your permission, I would like to focus first of all on the idea of "Orthodox philosophy", which in my opinion is so important, for at least three reasons: the rebirth of Orthodoxy in the secular and postmodern society; a right-minded understanding of the Orthodox civilisation; and of course the cultural importance of Orthodoxy even for the Western societies, which societies are based mainly on Catholic or Protestant identities. Talking about any orthodox philosophy could be something challenging and difficult but it is a moral duty for us to highlight the philosophical tasks of Orthodoxy. From this point of view I would like to ask you if you consider that the main characteristic of Orthodoxy could be metaphysics. And when I am saying "metaphysics" I especially make reference to the works of Holy Fathers of the Church such as Saint Maximus the Confessor.

Travis Dumsday: The issue of the relationship between Orthodoxy and philosophy is of course very old and very complicated. It is well-known that the dialogue with Greek philosophy (Aristotle, Plato, the Stoics, the neo-Platonists etc.) had an important set of roles to play in Church history, from the apostolic era onwards. These included roles in the understanding of Church dogma (e.g., the use of concepts like 'substance' and 'essence' in discussions at the early Ecumenical Councils) and in the defence of that dogma.

These facts apply historically to all of the main sub-disciplines of philosophy, including: ethics (the study of morality); epistemology (the study of knowledge and of related concepts like evidence and rationality); and, as you rightly note, metaphysics (the study of the nature and existence of things). In each case, the Holy Fathers made considerable use of Greek philosophical ideas.

That having been said, one of the interesting features of the relationship specifically between *Orthodoxy* and philosophy is the great diversity apparent in the Tradition. Orthodox thinkers have been influenced by a variety of philosophical schools (again, Aristotelian, neo-Platonic etc.), and there is no one philosophical school which could be considered as *the* official philosophy of the Orthodox Church. This internal diversity distinguishes Orthodoxy somewhat from Roman Catholicism, which has historically tended to treat Thomism as a sort of semi-official philosophical system.

Tudor Petcu: We know very well the evolution and the foundations of contemporary American philosophical approaches and, if I'm not wrong, I think by the American philosophy we may understand first of all analytic philosophy defined especially by the philosophy of language and different theories in logics which are, of course, essential for our philosophical background. I would also take into account the contemporary American political philosophy influenced especially by the idea of global justice, well-known because of Thomas Pogge. Given all I have mentioned above, what would be the place of Orthodox philosophy among these American philosophical approaches?

Travis Dumsday: It's certainly true that in the English-speaking world, the analytic approach remains the dominant way of doing philosophy. Being an analytic philosopher myself, I'm glad! Moreover, I think that Orthodox theology fits very nicely into analytic modes of thought. Analytic philosophy is characterized in part by its great emphasis on logic and clarity of argumentation. This focus on clear argument is absolutely evident in many of the Holy Fathers. Look for instance at some of the arguments employed by St. Athanasius or St. John of Damascus; indeed, just look at the overall structure of their major works. They are clearly organized and logically structured. So I think there is a natural sympathy between analytic philosophy and Orthodoxy.

That having been said, there are of course other major traditions in contemporary philosophy besides the analytic, and Orthodox thinkers cannot afford to ignore them. I think for instance of the continental, pragmatist, and Scholastic traditions. These too can be productive dialogue partners with Orthodox thought (in particular the Scholastic tradition).
I'm afraid I'm very poorly read in American political philosophy, so won't try to say anything about Prof. Pogge!

Tudor Petcu: I would be very interested to find out how you understand orthodox spirituality in North America, particularly in Canada, with its own heritage, traditions and particularities. Let's talk about for example the contributions of John Meyendorff to the evolution of American Orthodoxy and its scholarship. I wouldn't forget Seraphim Rose too, who is deeply appreciated especially in the Eastern Europe. So, tell me please: how would you characterise American Orthodoxy?

Travis Dumsday: God always works to bring good out of evil, and a wonderful example of this providential work can be found in the intellectual tradition sparked by the Russian diaspora. After the communist revolution of 1917, many important Orthodox intellectuals left for Paris and other major centres of scholarship, including, eventually, centres in North America. Many of the most important North American Orthodox scholars are the products of this diaspora, even if second-generation.

Of course, Orthodoxy was present in North America even before the communist takeover in Russia, as evidenced (for example) by the inspiring history of Orthodox missionaries in Alaska. And, living as I do in Alberta, I must mention too the huge waves of Ukrainian immigration to western Canada beginning in the late 19th century. That pre-revolutionary diaspora has also bequeathed a remarkable Orthodox heritage to Canada.

I am pleased to hear that Fr. Seraphim is being widely read in Eastern Europe. I have certainly benefitted from his writings (though I do disagree with some of his ideas, for instance his views on creationism).

Tudor Petcu: What does it mean for you to be an Orthodox Christian? And what does it mean for you to be an Orthodox philosopher in Canada?

Travis Dumsday: To be an Orthodox Christian is, in part, to have the amazing privileges of (a) access to the fullness of truth and (b) access to valid sacraments. Other Christian bodies do of course teach a great deal of the truth, but it is mixed in with error. And while other Christian bodies do have some valid sacraments (notably baptism), only in the Orthodox Church can we confidently partake of the genuine body and blood of Christ in the eucharist.

I love the philosophical scene in Canada; we have an energetic philosophical community and a variety of philosophical associations that actively hold conferences etc.
On being an *Orthodox* philosopher in Canada, I do try to bring my faith to bear on at least some of my philosophical work. In fact I'm very fortunate to be teaching at an institution (Concordia University of Edmonton) which encourages the pursuit of distinctly Christian scholarship.

Tudor Petcu: One of the deepest spiritual ideas which impressed me much and influenced my thinking remains, and always will, "the hidden holiness", idea which belongs to Michael Plekon. I would like you to tell me: how do you understand the hidden holiness from your own orthodox perspective?

Travis Dumsday: To my discredit, I'm not very familiar with the works of Plekon.

Tudor Petcu: Which would be in your opinion the most important role of Orthodoxy in the postmodern and pragmatic world? Could there exist any common denominator between Orthodoxy and pragmatism?

Travis Dumsday: I think it would depend on the particular version of pragmatism under discussion. Personally I am sceptical of many ideas coming out of pragmatist schools of thought, but some versions are friendlier to Orthodoxy than others. For instance, the works of C.S. Peirce are (in large part) broadly compatible with many Orthodox ideas. By contrast, someone like Richard Rorty is very much antithetical to Christian ways of thinking. Something similar could be said of postmodernism; there are certain strands of thought in postmodernism that are compatible with Christianity and can even provide new insights to Christian thinkers (a point emphasized by the American philosopher James K.A. Smith), but much of it seems to me deeply problematic.

Tudor Petcu: Given your entire Orthodox experience, and I would say way of living, what is the most relevant and the deepest idea that we can find in Orthodox spirituality and theology? Would it be correct to say that Orthodoxy represents the highest and the deepest way of living?

Travis Dumsday: I think the deepest idea we can find in Orthodox theology is in fact the deepest idea we can find in Christianity generally: God loves us so much that He sent his only-begotten Son to die for us. That is absolutely mind-blowing, and if a person really believes it, it will have a profound impact on every area of his or her life.

Tudor Petcu: If you should recommend one book, just one book, to someone who wants to better understand Orthodox spirituality, which would it be?

Travis Dumsday: Tough question! I'm not really sure; however, I am fond of Metropolitan Anthony Bloom's little book titled *Living Prayer*. That's certainly a good place to start.