

Endogenous Growth Model With Financial Intermediation

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Abstract:

In this paper, we analyse the simplest possible three-dimensional model of endogenous growth to account for the relationship between financial intermediation and economic growth. In our setting, households maximize an interim utility function and firms maximize profit. Households can save money only through banks which offer firms investment loans. We show that under very general assumptions, investments realized by firms depend not only on savings accumulated by banks but also on financial intermediation technology $\varphi(\theta)$. Using mathematical methods of dynamical systems, we found stationary states of the system and study their stability.

Keywords: dynamic analysis, endogenous growth, financial intermediation, multiple equilibria.

1. Introduction

Analyses of the relationship between financial intermediation and economic growth gained in importance in the 1970s. According to them, the developed financial system (operating on the principles of a market mechanism) leads to a more optimal allocation of funds in the economy and boosts economic growth.

After 2007, the approach to the relationship between financial intermediation and the real economy changed. The events that took place during the crisis undermined the issue, proposed among others by King and Levine King [17, pp. 717-737], regarding the long-term relationship between the financial intermediation sector and the real economy. Since the Great Depression of the 1930s, the global market has been redefined: mechanisms, instruments and a surveillance system have changed. Despite these significant differences, the risk of the financial crisis did not diminish [34]. The emergence of complex financial instruments, protecting investors against the risk of losing their funds, limited the transparency of the entire financial sector and became one of the factors stimulating the crisis [31].

In the face of the globalization of financial markets, the lack of global control systems or the appropriate management mechanism may decide about the instability of the financial sector [28]. Financing the economy, understood as increasing the influence of the institution financial resources on the real economy, an increase in the number of financial instruments on the market with a complex structure, as well as deregulation and liberalization of markets have become a source of risk growth in financial markets and limited control of institutions responsible for supervision of the financial market.

The perception that banks are one of the key elements supporting long-term economic growth had to be redefined [37]. However, one of the most important assumptions could not be changed – the financial intermediation sector played, plays and will play one of the most important roles in the economy. It is widely accepted that the uncontrolled development of the financial intermediation sector has contributed to the largest financial crisis that has occurred on the market since the Great Depression. Disorders that were visible on the markets have threatened the prospects of balanced and long-lasting economic growth.

A significant part of scientific research indicates positive relationship between financial intermediation and economic growth [12]. Empirical research on individual industries, markets and comparative research will mainly indicate that an efficiently operating financial sector not only leads to lower transaction costs and information asymmetry, but also mobilizes savings and provides financing for the most promising investment projects.

At the end of the 1990s, a correlation was sought between the level of development of financial intermediation and economic growth. Cihak, Demirguc-Kunt and Levine [9] pointed to a positive correlation between economic development and size (assets and liabilities) and efficiency (interest margin and administrative costs) of the financial sector. Hannson and Jonung [14] have shown that the credit per capita in relation to GDP is not fixed and varies in specific time intervals. These studies have shown that the level of economic development of a given economy is negatively correlated with the role of the central bank and positively with the effectiveness of commercial banks. There is also a negative correlation between economic growth and the degree of concentration of the banking sector, as well as a positive correlation with the level of development of financial markets [25].

Khan and Senhadji's [16] empirical research in the area of dependence of financial intermediation and economic growth has shown that financial intermediation positively affects economic growth, however the strength of this influence depends on the choice of estimation methods, indicators and the period under consideration.

Theoretical concepts indicate the possibility of a negative impact of financial intermediation on economic growth. They have gained in importance after the financial crisis of 2007. They mainly take into account the role of excessive participation of financial intermediation, whose negative effects of functioning displace positive. Uncontrollable growth of the financial sector may also lead to an increase in the frequency of financial crises. Large fluctuations in the dynamics of economic growth may adversely affect long-term development prospects. Cecchetti and Kharroubi [8] showed that every unnaturally fast-growing industry sucks resources, such as physical capital and human capital, from the rest of the economy. In this situation, the marginal productivity of the large financial sector is lower than in the other sectors. The shift of large capital and labor resources to the sector representing financial intermediation inhibits the potential for economic development of a given economy. This process reveals the inefficiency of the market mechanism and increases the probability of a future crisis, within which the proportions between the financial sector and other sectors operating in a given economy would be balanced.

The results of empirical research describing the relationship between financial intermediation and economic growth are not conclusive. They indicate both the positive and the flawed effects of the development of the financial system due to the increase in the availability of financing expenditure on education and the greater resilience of households to external shocks due to the increase in the level of banking. OECD research shows that the further development of the financial sector, measured by the

value added of the financial sector in relation to GDP and the amount of loans granted, would contribute to a decline in the income of less affluent households. At the same time, this phenomenon would accelerate the growth of the richest income, thus increasing income inequalities in the economy.

The choice of this subject is justified from the point of view of literature in this area, which is available on the market. The first article that drew attention to the issue of the diversity of the transmission of financial mechanisms to the real sphere and their impact on economic growth was the article by Pagano [28]. Mainly theoretical literature describing the relationship between financial intermediation and economic growth focuses on the effects of financial intermediation and the entire financial sector on human capital (De Gregorio and Kim [10], Bucci and Marsiglio [6]). Attention is also paid to the effects seen in the accumulation of physical capital (Trew [34], [35], Bucci and Marsiglio [7]). Recent empirical results are trying to explain the ambiguity of the relationship between financial intermediation and economic growth seeking a justification for such results in the form of non-linearity and non-monotonicity of the relationship between these variables) Cecchetti and Kharroubi [8], Law and Singh [20], Arcand et al. [2]).

This work aims to fill the research gap consisting in theoretical description of relations between financial intermediation and economic growth.

2. Model

Let us consider the economy with two sectors: industrial and financial. The industrial sector gives the product Y described by the neoclassical production function in the Cobb-Douglas form

$$Y(t) = K(t)^\alpha L_I(t)^{1-\alpha}, \quad (1)$$

where $K > 0$ is physical capital stock. The labour $L > 0$ is employed in industrial sector, $L_I > 0$, and in financial sector, $L_F > 0$, such that $L_I + L_F = L$.

We assume that the labour grows with a constant rate $n \geq 0$, i.e.,

$$\dot{L}(t) = nL(t). \quad (2)$$

As the both L_I and L_F grows with the same rate n , the shares of employment in financial and industrial sectors are

$$\theta = \frac{L_F}{L}, \quad 1 - \theta = \frac{L_I}{L}, \quad (3)$$

respectively.

The accumulation of capital is governed by

$$\dot{K}(t) = Y(t) - \delta K(t) - C(t), \quad (4)$$

where C is consumption and δ is the rate of capital depreciation.

Let us introduce new variables

$$y(t) = \frac{Y(t)}{L(t)}, \quad k(t) = \frac{K(t)}{L(t)}, \quad c(t) = \frac{C(t)}{L(t)}. \quad (5)$$

Then the equation for capital accumulation in these new variables is

$$\dot{k}(t) = k(t)^\alpha(1 - \theta)^{1-\alpha} - (n + \delta)k(t) - c(t). \quad (6)$$

Now, consider the households which maximize their utility from consumption in infinite time horizon. The intertemporal utility function is

$$U = \int_0^{+\infty} \exp(-\rho t) u(c(t)) dt, \quad (7)$$

where $\rho > 0$ is the subjective discount rate. The utility function is

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \sigma > 0. \quad (8)$$

The maximization problem has the form

$$\max_c(c(t)) \quad (9a)$$

$$\text{subject to: } \dot{k}(t) = k(t)^\alpha(1 - \theta)^{1-\alpha} - (n + \delta)k(t) - c(t). \quad (9b)$$

Using the Pontryagin maximum principle we obtain

$$\dot{c}(t) = \sigma[\alpha k(t)^{\alpha-1}(1 - \theta)^{1-\alpha} - (n + \delta + \rho)]c(t). \quad (10)$$

The financial institutions takes the savings from households and transfer it to firms. The relation which links investment and savings was proposed by Eggoh and Villieu (2014) in the form

$$\dot{K}(t) = \varphi(\theta)\dot{B}(t), \quad (11)$$

where B represents the level of household savings in the form of deposits accumulated in banks, and $\varphi(\theta)$ is the technology of intermediation with the following properties

$$0 \leq \varphi(\theta) \leq 1, \quad \varphi'(\theta) > 0, \quad \varphi''(\theta) < 0, \quad \varphi(0) = 1. \quad (12)$$

Similarly, as for capital K and C , let us introduce the variable $b = B/L$. Then we obtain

$$\dot{b}(t) = \frac{1}{\varphi(\theta)}[k(t)^\alpha(1 - \theta)^{1-\alpha} - \delta k(t) - c(t)] - nb(t). \quad (13)$$

Finally, we obtain the three-dimensional dynamical system

$$\dot{k}(t) = k(t)^\alpha(1 - \theta)^{1-\alpha} - (n + \delta)k(t) - c(t) \quad (14)$$

$$\dot{c}(t) = \sigma[\alpha k(t)^{\alpha-1}(1 - \theta)^{1-\alpha} - (n + \delta + \rho)]c(t)$$

$$\dot{b}(t) = \frac{1}{\varphi(\theta)}[k(t)^\alpha(1 - \theta)^{1-\alpha} - \delta k(t) - c(t)] - nb(t)$$

This system has two critical points. The first critical point is

$$k_1^* = b_1^* = c_1^* = 0. \quad (15)$$

The second critical point is

$$\begin{aligned}
k_2^* &= \left[\frac{n+\delta+\rho}{\alpha} (1-\theta)^{\alpha-1} \right]^{\frac{1}{\alpha-1}} \\
c_2^* &= (k_2^*)^\alpha (1-\theta)^{1-\alpha} - (n+\delta)k_2^* \\
b_2^* &= \frac{1}{n\varphi(\theta)} [(k_2^*)^\alpha (1-\theta)^{1-\alpha} - \delta k_2^* - c_2^*]
\end{aligned} \tag{16}$$

The phase space of the system for economic meaning of model variables is restricted to

$$P = \{(k, b, c): k > 0, b > 0, c > 0\}. \tag{17}$$

Let us consider the local stability of the critical point. Its stability is characterized by the linearization matrix evaluated at this critical point p^* .

$$A = \begin{bmatrix} \rho & -1 & 0 \\ \sigma(\alpha-1)(n+\delta+\rho)\left(\frac{n+\delta+\rho}{\alpha} - n - \rho\right) & 0 & 0 \\ \frac{n+\delta}{\varphi(\theta)} & -\frac{1}{\varphi(\theta)} & -n \end{bmatrix} \tag{18}$$

Then its characteristic equation is given as

$$\begin{aligned}
\det[A - \lambda I] &= (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) \\
&= \lambda^3 - (tr A)\lambda^2 + (tr A^2 - (tr A)^2)\lambda - \det A = 0,
\end{aligned} \tag{19}$$

and the eigenvalues of the linearization matrix are real and equal to terms on the diagonal

$$\begin{aligned}
\lambda_1 &= -n \\
\lambda_2 &= \frac{\rho-4\sqrt{\rho^2-4(\alpha-1)(n+\delta+\rho)\left(\frac{n+\delta+\rho}{\alpha}-n-\rho\right)}}{2} \\
\lambda_3 &= \frac{\rho+4\sqrt{\rho^2-4(\alpha-1)(n+\delta+\rho)\left(\frac{n+\delta+\rho}{\alpha}-n-\rho\right)}}{2}
\end{aligned} \tag{20}$$

where $\rho^2 > 4\sigma(\alpha-1)(n+\delta+\rho)\left(\frac{n+\delta+\rho}{\alpha}-n-\rho\right)$. This equation is always true when $n > 0$, because for $n > 0$, we have $4\sigma(\alpha-1)(n+\delta+\rho)\left(\frac{n+\delta+\rho}{\alpha}-n-\rho\right) < 0$.

In this case the unique critical point has all eigenvalues real, two with negative values (λ_1, λ_2) and one with a positive value (λ_3) when following conditions are satisfied

$$\frac{n+\delta+\rho}{\alpha} - n - \rho > 0. \tag{21}$$

The above equation can be reduced to a simpler version

$$(1-\alpha)(n+\rho) + \delta > 0. \tag{22}$$

This equation is always true for any parameters α, ρ, n and δ that meet the assumptions of the model. Therefore, the phase space P is a product

$$P = P_{stable} \oplus P_{unstable}, \quad (23)$$

where the stable submanifold is two-dimensional ($\dim P_{stable} = 2$) and the unstable submanifold is one-dimensional ($\dim P_{unstable} = 1$).

Because we are interested in the critical points in the domain P , all their coordinates k^* , b^* , c^* should be strictly positive. This is guaranteed if following conditions are satisfied

$$\begin{aligned} n + \rho + \delta &> 0 \\ \left(\frac{1-\theta}{k}\right)^{1-\alpha} &> n + \delta \\ nk - c &> 0 \end{aligned} \quad (24)$$

3. Saddle-node Bifurcation

In this section we study local bifurcation in our system. Let us restrict the analysis to the bifurcation of codimension 1.

The characteristic equation has the form

$$\lambda^3 - (\text{tr } A)\lambda^2 + [(\text{tr } A)^2 - \text{tr } A^2]\lambda - \det A = 0 \quad (25)$$

The coefficients of characteristic equation can be expressed in terms of eigenvalues ($\lambda_1, \lambda_2, \lambda_3$). The quantities are given by

$$\text{tr } A = \lambda_1 + \lambda_2 + \lambda_3 \quad (26a)$$

$$\text{tr } A^2 - (\text{tr } A)^2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \quad (26b)$$

$$\det A = \lambda_1\lambda_2\lambda_3. \quad (26c)$$

In our case these quantities assume the following form

$$\text{tr } A = \rho - n \quad (27a)$$

$$\text{tr } A^2 - (\text{tr } A)^2 = -n\rho + \frac{\rho^2}{4} - 4(\rho^2 - 4\sigma(\alpha - 1)(n + \delta + \rho) \left(\frac{n + \delta + \rho}{\alpha} - n - \rho\right)) \quad (27b)$$

$$\det A = -n \left(\frac{\rho^2}{4} - 4(\rho^2 - 4\sigma(\alpha - 1)(n + \delta + \rho) \left(\frac{n + \delta + \rho}{\alpha} - n - \rho\right))\right) \quad (27c)$$

In the dynamical system with continuous time the local bifurcation appears when real part eigenvalues $\lambda(p)$ crosses zero as we change parameter p . Let us denote p^* a critical value of the bifurcation parameter. It could be useful to distinguish two generic cases:

1. when real part of eigenvalues crosses zero: $\lambda(p^*) = 0$, the system undergoes saddle-node bifurcation.
2. when real part of complex and conjugate eigenvalue $\lambda(p) = \zeta(p) \pm i\omega(p)$ crosses zero then the system undergo the Hopf bifurcation.

The Hopf bifurcation is a special type of bifurcation, which consists in the appearance of limit cycles as a result of bifurcation from a stable singular point.

Proposition 1. *The saddle-node bifurcation arises if and only if $\det A = 0$.*

There are two possibilities:

1. The first case is trivial as the employment is stable $n = 0$.

2. In the second case $\frac{\rho^2}{4} - 4(\rho^2 - 4\sigma(\alpha - 1)(n + \delta + \rho))$, but this equation is contrary to the assumptions. Thus, we deal with saddle-node bifurcation only when $n = 0$

The case of complex eigenvalues is the most interesting one from the point of view of the dynamical system theory.

Proposition 2. *If there exist a pair of complex conjugate eigenvalues λ_2 and λ_3 , the system oscillates with vanishing amplitude if the real parts of λ_3 and λ_2 are negative and $\lambda_1 < 0$.*

In this case, the system is locally asymptotically stable.

Proposition 3. *The Hopf bifurcation gives rise to the limit cycle either attractive (supercritical) or repulsive (subcritical) if and only if $\det A = [\text{tr } A^2 - (\text{tr } A)^2](\text{tr } A)$ and $\text{tr } A^2 - (\text{tr } A)^2 > 0$.*

One can conclude the Hopf bifurcation does appear as the condition $\text{tr } A^2 - (\text{tr } A)^2 > 0$ is satisfied for our model. But in our case $\text{tr } A^2 - (\text{tr } A)^2 = -n\rho + \frac{\rho^2}{4} - 4\left(\rho^2 - 4\sigma(\alpha - 1)(n + \delta + \rho)\right)\left(\frac{n + \delta + \rho}{\alpha} - n - \rho\right) < 0$. Thus, Hopf's bifurcation does not appear in our model.

4. Conclusions

In this paper, we re-examine the relationship between financial intermediation and economic growth from a theoretical perspective. Our model is given by the simplest possible three-dimensional model of endogenous growth. In our model, the households optimize the utility from consumption and their savings are transformed into investment through the bank system.

Despite the fact that financial intermediation is one of the most important elements of modern economies and a lot of empirical research in this area, there is little theoretical work showing the dynamic relations between financial intermediation and economic growth. This work aims to fill this research gap and to thoroughly analyse the channels regarding the development of financial intermediation and its impact on economic growth. In order to visualize the relations that occur between financial intermediation and economic growth, we will analyse a very simple endogenous model of economic growth.

Our results are the following:

- The dynamics of the model can be represented as a three-dimensional dynamical system in variables: a ratio of consumption to capital, a ratio of bank deposits to capital and the level of employment in the banking system.
- The saddle-node bifurcation was found in the model. Due to this bifurcation, the saddle critical point is created toward which the system evolves along the stable optimal path.
- We showed that saddle-node bifurcation arises when $n + \delta + \rho = 0$ (a collision and disappearance of two equilibria in our system, this occurs when the critical equilibrium has one zero eigenvalue). Movement along the path of the equilibria is restricted to movement along the saddle, with bifurcation along that path occurring at the origin
- Additional aspects of financial intermediation process such as financial intermediation technology $\varphi(\theta)$ and employment in banking system θ should be taken into account in order to make economic growth more predictable.

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