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The Fluttering of Autumn Leaves: Logic, Mathematics, and Metaphysics in Florensky's *The Pillar and Ground of the Truth*

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Abstract:
Difficulties in understanding Pavel Florensky's work *The Pillar and Ground of the Truth* are daunting due in large part to its methodical transgressing of identities: between disciplinary boundaries (his work drawing freely from philosophy, theology, logic and mathematics, art history, linguistics, and philology); between literary identities, as he fluidly shifts between literary criticism, logical proof, poetic discourse, and philosophical dialectics in his own writing; as well as in collapsing identities between concepts that appear to be binary and incompatible. Nor does his work proceed in the developmental and synthetic manner of German Idealism, aiming toward higher and increasingly more hegemonic syntheses, but instead through emphasizing discontinuity, otherness, and antinomy. Important insights can be gained into both the foundations and the broader importance of his work by seeing that these difficulties are intentionally generated by the author, and arise largely from his philosophical commitments in logic and mathematics, and above all his attempt to go beyond the limits of the Aristotelian principle of identity through outlining a more fundamental principle of identity influenced as much by Heraclitus and the ascetic theology of the Eastern Church as it is by Georg Cantor's research into the mathematics of infinity and by the celebrated Russian School of Mathematics, of which Florensky was himself a founding member.

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1. Two Worlds

Pavel Florensky's *The Pillar and the Ground of the Truth* [4] (hereinafter *Pillar*) is surely one of the most unusual books of philosophy published in the twentieth century. More often than not, it produces in the reader a consternation that has caused many to reject it altogether after a few glances, thinking it an example of the aestheticism and even decadence that has come to be associated with Russian Symbolism. This stigmatization is both unfortunate and unjust, for it is a work of great logical, mathematical, and philosophical rigor as well as a source of deep spiritual insight. Moreover, one of its primary claims is that the formal rigor of logic and mathematics is ontologically rooted – not just applicable to the real, but of one piece with being itself. And another of its claims is that spirituality does not concern some rarified dimension separate from empirical reality, divorced from the human body and natural science and works of art, but that it extends into and illumines every aspect of life; it does not inhabit a world unto itself. Transcendence and immanence, visible and invisible, are not just "two worlds," but ultimately two aspects of one world. Like Heraclitus and Parmenides before him, Florensky seeks to show that "it is wise to agree that all is one," *hen panta*.

Thus, for example, the sky-blue color which he chose for the cover of the book (and to which he devotes a seventeen page excursus, rivaling Melville's chapter exploring the color white) is a visual representation of one of his central themes, "Sophia" or Divine Wisdom – the theme for which the book is best known, and which is perhaps better approached only after some of its more basic concepts have been mastered. So too, the book is written not as a series of chapters, but a sequence of twelve "Letters" written to a close, but unspecified friend, each of which (like a Japanese *haiku* poem) begins with an evocation of the natural surroundings that indicates the season of its writing and reflects a mood that attunes what follows. Each chapter is headed by a different graphic vignette, depicting some curious object or action, and drawn from peculiar and esoteric sources, along with rather cryptic sayings that resonate with the vignette in some indefinable manner. Yet the text that follows is never something merely "aesthetic." It might just as likely involve a discussion of scientific findings, of world mythology, of mathematical analysis or philosophical logic, of comparative linguistics and etymology, of theological controversies from the fourth or eleventh centuries, and of course perennial philosophical difficulties – all interwoven and mutually illuminating one another, all advancing the investigation which the book undertakes in a rigorous and carefully crafted manner. But this lush and lavish variety is not put forth merely to display his extraordinary intellect and prodigious learning (which have made it fashionable to compare him to Leonardo da Vinci) but to show how each of these disciplines leads to the same understanding of his great themes – and thus that, say, mathematics and theology and ethics might be not just interconnected, but properly understood, different languages for the saying the same things. If all things are one, then any starting point will lead to the same conclusion. And demonstrating this is not extraneous to the subject matter – a *tour de force* of intellectual virtuosity – but part of the argument itself, one that must be experienced by the reader, rather than merely asserted by the author.

At first glance, however, this bewildering juxtaposition of writing styles from the lyrical to the logical; of disparate disciplinary approaches and concepts; and of four different literary formats that includes two sizes of type and some 1057 footnoted elucidations occupying one quarter of the text, along with 15 often lengthy appendices – all this seems disconnected and discontinuous, requiring strenuous leaps of understanding. And this is just as Florensky intended, for the idea of *discontinuity* is itself one of the key themes of the book, and the author attempts to lead the reader to this insight precisely through the employment of "discontinuous thoughts," as he characterizes his writing – a mode of exposition that he shares with Kierkegaard and Nietzsche before him, and Heidegger after him, not to mention the "dark," aphoristic style of his master, Heraclitus. Moreover, the author goes on to announce that he will be proceeding "without system, only placing a signpost here and there" with the expectation of arriving only at "schemata" and "fragments" (*Pillar*, p. 13). Visible and invisible, same and other, heaven and earth, transcendence and immanence are not two worlds but one. And yet it takes a leap, a discontinuous trajectory, to realize this – to experience the epiphany of the one within the many, of the heavenly within the earthly – and the most demanding and rigorous philosophical work to lead to the brink of this leap – and to convince the reader not to lose heart at the edge of the precipice. How, then, can genuine rigor of thought be combined with the kind of intuitive, and indeed existential, demands that are more commonly expected in poetic and religious writing?

"Letter One," itself subtitled "Two Worlds," begins early in the morning, on that first day in which it has become evident that summer is over and "something new" is in the air. "Golden leaves whirled over the ground in serpentine, wind-driven eddies," fluttering like butterflies. "The air was filled with the cool aroma of autumn, the smell of decaying leaves, a longing for the distances." "One after another, leaves were falling to earth... describing slow circles in the air as they descended to earth." "How good it was," he exclaims, "how joyous and sad" was the "sight of these fluttering leaves." "Autumn leaves keep falling, without interruption," the author continues, and as he watches them he reflects on friends who have come and gone, he reflects on temporality and death: "Everything whirls. Everything slides into death's abyss." Is there a center, he asks, toward which all these whirling trajectories would point – recalling perhaps the transcendent "point at infinity" that allows the infinite number of points on a given plane to converge around the Riemann Sphere, but more explicitly evoking that Center toward Whom is drawn "the whole course of events, as the periphery to the center," and toward Whom "converge all the radii of the circle of the ages," the Center described by the nineteenth century Russian saint, Theophan the Recluse? Or must we concur with the sad wisdom of Pliny the Elder, that since "in life everything is in a state of unrest," then "the only certain thing is that nothing is certain and that there is nothing more miserable or arrogant than man" (*Pillar*, p. 12). How to draw together the two worlds of Parmenides, the realms of being and appearance, time and eternity, finite and infinite? How to affirm with Heraclitus that within the change and flux and fluttering upon which he dealt at such length, there is yet a unity that draws together all things, and thereby allows us to gather them together in thought and language?

2. Noumena and Numbers

In his autobiographical account of his boyhood in the Caucasus Mountains, Florensky describes at length the way in which nature everywhere spoke to him, manifested to him its inner life – described what he called "the unusual yet sweetly known and familiar revelation from native deeps" that he found all around him in the rugged canyons and gentle seaside of his native Georgia, and that continued to motivate his studies in mathematics, science, philosophy, and theology even to his final days, during which he occupied himself by studying algae while confined in the brutal Solovetsky Monastery *Gulag* on the White Sea, just below the Arctic Circle.[1] It was not physical nature as such that enthralled and enchanted him, but what he later called the "*Empyrean*" – the divine or heavenly – manifest in the empirical or earthly, and which he associated with the inner reality of things, their noumenal character as they were rooted in the Divine, a rootedness that he came to call "Sophia" or Divine Wisdom. And it was in the mysterious character of the symbol that he found the locus of this conjunction between the two worlds:

> All my life I have thought, basically, about one thing: about the relationship of the phenomenon to the noumenon, of its manifestation, its incarnation. It is the question of the *symbol*.[2]

Not surprisingly, then, Florensky came to see his great nemesis in the philosopher whom, perhaps more than any other, he felt had led modern thought astray:

> The Kantian separation of noumena and phenomena (even when I had no suspicion of the existence of any one of these terms: 'Kantian,' 'separation,' 'noumena' and 'phenomena') I rejected with all my being.[3]

Instead, Florensky felt strongly drawn to the tradition of Platonism, which he saw as joining together these two worlds, of showing how the visible made manifest the invisible, and how the invisible shines through the visible.[4] It was, then, to a strongly realist approach to mathematics (in the Platonic sense of "realism" that sees mathematics as ontological, rather than empirical or psychological or constructivist) that Florensky was drawn in his earliest studies, and above all to the investigations of Georg Cantor, whom he thanks for his own understanding that "the number is therefore a prototype, an ideal schema, a primary category [both] of thought and of being" [3], p. 195.

"For me," wrote Florensky to his mother at the age of 18, "mathematics is the key to a world view... for which there would be nothing so unimportant as not to be worth studying and nothing that was not linked to something else" (cited in [7], p. 27). Several years later, he was to write:

> My studies of mathematics and physics led me to acknowledge the formal possibility of theoretical foundations for a religious world view for all humanity (the idea of discontinuity, the theory of functions, numbers.) (cited in [7], p. 36f).

Most of Florensky's earliest papers were on mathematics, and a recent critic, S. S. Demidov, has maintained that

> without [an] understanding of the significance of mathematics in his method of understanding the world, outside the frame of his opinions on the place of mathematics in the Universe it is impossible adequately to evaluate either his method or his philosophical views.[5]

Florensky was fortunate to study with one of the great Russian mathematicians of the early twentieth century, Nikolai Bugaev, and he is considered by a recent study in English to be (along with Nikolai Luzin and Dimitri Egorov), one of the “trio” of founders of the Russian School of Mathematics. In this paper, then, mathematics will serve as a key for understanding some of the central concepts of his greatest work.

The *Pillar and the Ground of the Truth*, consistent with the very task it takes upon itself, can legitimately be read in many ways. It can be approached as a sustained inquiry into the theology of the Christian Trinity, perhaps one of the most important since Chalcedon. It can be read as one of the great philosophical attempts to resolve philosophy’s perennial problem of the One and the Many, the Same and the Other. It can be read, as its own subtitle suggests, as an “Orthodox Theodicy,” justifying the ways of God to man, by showing the necessity of asceticism and suffering, the ontological grounds of sin, and even the possibility of what he calls “Gehenna” in the ceaseless striving of “bad infinity.” But it can also be approached from the direction of formal reasoning, mathematics and logic, as will be done in this paper. From this perspective, it can be read as a sustained assault upon the primacy of the law of identity – a principle that has been taken since Aristotle as the foundation of formal reasoning – an assault that paradoxically employs important concepts of mathematics and logic themselves, such as the concepts of actual and potential infinity, discontinuous functions, the recurrence of antinomies, and the problem of irrational and transcendental numbers. Yet paradoxically, it is only the primacy of the law of identity that Florensky seeks to overthrow, not the law itself. Indeed, he seeks to show that the law of identity is grounded in something deeper and more basic than logic. Just as Heraclitus and Parmenides believed, it is grounded in the nature of being itself when it is understood according to the mode of truth that is proper to it.

3. The Law of Identity and its Limitations

It will, perhaps, be useful to present at the beginning a very abstract formulation of Florensky’s claims concerning the law of identity. Florensky argues that there are higher and lower versions of the law of identity, one that is ultimately empirical and psychological (and which has been traditionally embraced by logicians) and the other reflecting an ontological understanding, a radically realist understanding, whereby the knower in a most important sense becomes the known, where A=A only by means of becoming not-A.[6] Here, the term identity applies to the relation of knower to known, of thinking to being, and not the relation of the knower to himself. The knowing self (A=A, which for Florensky is ultimately I=I) must go out of itself, leave itself behind and unite with the known, in order to know and in order to be itself in more than an abstract sense. And conversely, the not-A that is known, can be known only within this unity of knowing: not-A must become A.

A=A, Florensky argues, is first of all numerical unity, and not simply generic or specific unity. Yet this numerical unity cannot be found in a thing, which exhibits only generic identity, but only in the person who is *himself* self-forming, self-realizing, self-creating. The thing, in contrast, can never be strictly speaking one, for it is merely a member of a larger unity – even if it happens to be the only member. Yet pure self-positing, in the Fichtean sense, is something purely empty, abstract, and ultimately negative. The “this-here-now,” immortalized in the first chapter of Hegel’s *Phenomenology of Spirit*, is nothing more than the negation of every other this, here, and now, a defensive or combative vacuum that indeed defines itself as a self-identity, but only in an abstract and purely negative way. “In excluding all the other elements, every A is excluded by all of them, for if each of these elements is for A only not-A, then A over against not-A is only not-not-A”

(*Pillar*, p. 23). But how, and on what basis, could A go beyond its identity to become one with not-A, I with not-I? How can the self go beyond itself to become one with the other: how does the merely psychological self-identity of self-assertion become the ontological identity that is proper to a person? Here, logic and ontology merge with theology, for the A that is A through becoming not-A is an A that is able to love, and love in this radical sense, Florensky argues, can only be realized through a kind of *ascesis* of the self-contained self, resulting in an openness to a mystical identity with an eternal reality whose own very being consists in a dynamic of unity with otherness. And the logical and mathematical principles with which Florensky seeks to undertake this philosophical journey are the concept of discontinuity; the contrast between actual infinity and potential infinity, along with the Absolute Infinity first discussed by Cantor; the antinomies of rationality, and thus the contradictions to which the lower law of identity must lead; and the contrast between generic identity and numerical identity. We will, then, take the last of the principles first, proceeding one by one through the other three until a point is reached at which a brief overview of Florensky's logical-mathematical critique and correction of the law of identity becomes possible.

4. Modes of Identity

Florensky argues that the neglect and misunderstanding of numerical identity extends back to Latin scholasticism and its logic of terms. Examining the logical works first of Thomas Aquinas and then Francisco Suarez, Florensky finds three modes of identity enumerated: generic, specific, and numerical *(generice, specifice, numerice).* That is, identity is understood as the negating of division according to genus, according to species, and according to number. It is as if, he argues quoting Suarez, a state of contraction *(status contractionis)* can be observed at work here, in which diversity of genus and species is progressively negated, and finally the size of the class itself is contracted into a singular class: the individual Socrates understood as no more than a class with only one member. But this kind of understanding of identity "remains limited to the category of things," leaving us with merely an impersonal entity that is no more than the shrunken remnant of its own tribe, a general concept identified with itself as a singular class. Strictly speaking, Florensky argues, this is not yet numerical identity at all – not truly one, but still essentially generic and general. For true self-identity to be possible, there must be something else entirely than such a "gradual evolution" from genus to species to the elimination of one member after another until there is only one left, a progression that can never yield more than conceptual, and external, identity. Rather, there must be a break, something new altogether: there must be the self-positing that is possible only from within, and thus only for a person who is not, nor cannot be, subordinated to any class at all (*Pillar*, pp. 365–368). The only beings capable of being numerically (rather than generically of specifically) identical with themselves are persons: "the source of the idea of numerical unity must be sought in the self-identity of consciousness" (*Pillar*, p. 60). For

> concrete individuals possess *creativity,* are capable of creating absolute, unforeseen relations, which are not part of any group, no matter how large, of already existing relations (*Pillar*, p. 374).

Rejecting the gradualism of a smooth, continuous "contraction" of class membership that stays within the realm of things and their properties, Florensky engages here a *discontinuity* that moves beyond thingness altogether, emerging into the identity of a realm of relations that cannot be categorized and grasped through rationality at all, yet which before all concepts and rationality is always already identifying itself.

> A thing is characterized through its *outer unity*, i.e., through the unity of the sum of its features, while a person has his essential character in an *inner unity*, i.e.in the unity of the activity of self-building... Therefore, the identity of things is established through the identity of concepts, while the identity of a person is established through the unity of his or her self-building or self-positing activity (*Pillar*, p. 59).

5. The Need for Discontinuity

Florensky argues that numerical identity is founded on consciousness, i.e. on the self-establishing reflexivity that is exclusively characteristic of persons. But this would mean that the law of identity, A=A, is really grounded in self-identity, I=I. Yet so far, the I=I is confined to simple self-positing: "I am I" means nothing more here than I am not this not-I, nor that not-I, nor yet another not-I, continuing unto a kind of infinity – the potential infinity or endlessness that Florensky argues characterizes the futility of mere self-identity in its various modes (and about which more will be said later.) I=I is sheer negation, and although it yields an actual (as opposed to merely conceptual) self-identity, it is purely negative, and thus is itself a kind of prison of self-affirmation and self-assertion. At the same time, it lacks any positive content of its own, beyond the negation that is entailed in self-assertion. Florensky describes this powerfully in a passage that those unaccustomed to the idea of linking thoughts in logic, metaphysics, psychology, and theology may find somewhat surprising:

> The law A=A becomes a completely empty schema of self-affirmation, a schema that does not synthesize any real elements, anything that is worth connecting with the "=" sign. "I=I" turns out to be nothing more than a cry of naked egotism: "I!" For where there is no difference, there can be no connection. There is therefore only the blind force of stagnation and self-imprisonment, only egotism. Outside of itself, I hates every I, since for it this [other] I is not-I; and hating, I strives to exclude this I from the sphere of being. <...> Thus, since the naked "now" is a pure zero of content, I hates the whole of its content, i.e., the whole of its life. I turns out to be a dead desert of "here" and "now" (*Pillar*, p. 23).

To escape from this "self-imprisonment," something radical must intervene, something incommensurate with the monadic self-positing of the I. It would have to break the bonds of the Cartesian *cogito*, which seeks in futility to transcend the bubble of solipsism through concepts alone. And it would also need to be more radical than the Hegelian *Aufhebung* which, even as it gradually raises the level of development, still evolves dialectically along an epistemological and ontological continuum, seeking otherness only to assimilate it into an expanded self-identity. There must be a second discontinuity, a leap traversing an abyss that is even more radical than the first one that led from thinghood to personhood – a *discontinuity* that would lead the self beyond the prison-walls of its own self-assertion (I=I), and thus would lead the law of identity itself beyond the monadism of A=A. Somehow, I must be more than I, and A more than A. And it is here that Florensky's great theme of discontinuity, mentioned already at several points above, assumes decisive importance. If the soul is to ascend beyond self-affirmation, if it is to find life in a "higher, spiritual law of identity, rather than the "lower, fleshly law of identity" which confines it, then this must be "attained not through gradual approach, not through continuous development, but through discontinuous rejection of selfhood" (*Pillar*, pp. 224f). As Kierkegaard had also seen clearly in his *Concluding Unscientific Postscript*, Truth cannot be attained through the bad infinity of what he called an endless "approximation process."

Florensky was always grateful to have studied with the great mathematician Nikolai Bugaev during his first semester at Moscow University. Bugaev sought to build on Cantor's work in set theory, his work on transfinite numbers, and his analysis of the "continuum," as a set of points – while himself pursuing research into the mathematics of discontinuous functions – in order to develop a critique of what he believed were the deterministic implications of the concept of continuity, a concept he saw as dominating the mathematical and scientific work of his time. For example, if every continuum is in fact an infinite set of discrete points, then discontinuity is more fundamental than continuity, an insight that he saw as important not just mathematically, but metaphysically as well. As Bugaev had written in 1897, "discontinuity is a manifestation of independent individuality and autonomy. Discontinuity intervenes in questions of final causes and ethical and aesthetic problems" [6], p. 68. Florensky, then, took delight in these famous lectures of

Bugaev, which linked the mathematics of discontinuous functions with “excursions into psychology, into philosophy and ethics,” an approach upon which Florensky himself was to build so richly ([7], p.27). The concept of discontinuity continued to be crucially important for Florensky throughout this work, and his undergraduate dissertation (for which he received the highest marks) was entitled, “On the Characteristics of Flat Curves as Loci for Breaks in the Continuum.”

But Florensky carried both the mathematics and the metaphysics of discontinuity beyond his teacher Bugaev. He saw the principle of continuity as the calamitous “governing principle” of nineteenth century thought as a whole, and he believed it was vital to overcome its dominance, which manifested itself in areas as diverse as Marx’s philosophy of history, the uniformitarian philosophy of Lyell in geology, and Darwin’s view of evolution as developing from gradual small changes. “The cementing idea of continuity,” he argued, “brought everything together in one gigantic monolith” ([6], p. 88). Subsequent thought has in fact, as he anticipated, vindicated Florensky on this point, from the post-modern critique of meta-narratives such as Marx’s, to the discovery of the role of chaos in meteorology and other earth sciences, to the realization of the role of mutations in biology, to the paradoxes of discontinuity in quantum mechanics, to the notion of paradigm shifts in the history and philosophy of science, but at the time the assumption of continuity and gradualism (perhaps a last manifestation of the “great chain of being” assumed in medieval thought) was dominant and everywhere taken for granted. As Florensky put it,

> the idea of continuity, making these transitions, took possession of all disciplines from theology to mechanics, and it seemed that anyone who protested against its usurpations was a heretic ([6], p 88).

Nevertheless, Florensky countered,

> inspiration, creativity, freedom, *ascesis*, beauty, the value of the flesh, religion, and much else... stands outside the methods and means of scientific research [as it is currently practiced], for the fundamental presupposition of such methods and means is, of course, the presupposition of connectedness, the presupposition of continuity, gradualness (*Pillar*, p. 94).

Yet Florensky sees this bondage to continuity and un-freedom as simply reflecting the limitations of nineteenth century science and mathematics, even as this presupposition was already being left behind through more recent discoveries that pointed instead to the primacy of discontinuity (*Pillar*, pp. 485f; 574).

Thus, both of the first two letters of *The Pillar and Ground of the Truth* revolve around one of the greatest of all discontinuities – the discontinuity between life and death. The first letter, discussed already, focuses not just on the melancholy of change and the transitory character of life, but more fundamentally upon the reality of death that underlies them. The endless whirling and fluttering of autumn leaves, “one after another,” suggests a kind of slow, spiritual death: the bad infinity of one sin after another, one petty baseness or inattention or cruelty after another scarring the soul, and “gradually crippling it.” And

> one after another, one after another, like the leaves of autumn, those people whom our heart has come to love forever whirl above the dark chasm. They fall, and there is no return, no possibility of embracing the feet of each of them” so that “now between me and them lies an abyss.

This abyss and chasm of death – this discontinuity between life and death that radically breaks with the continuity of decline – poses at the same time the thought of renewal and new life. “It appears that the soul has a foretaste of resurrection in this fluttering,” and in this “fragrance of faded aspen groves” (*Pillar*, p. 11). Just as the ceaseless fluttering leaves evoke the longing for a center, so too do the endless truths that correspond to our boundless curiosity suggest our need for a single, central truth. Here we discover within ourselves a hunger not just for

> the particular and fragmented human truths, which are unstable and blown about like dust chased by the wind over mountains, but [for] total and eternal Truth, the one Divine Truth, the radiant and celestial Truth (*Pillar*, p. 12).

And as will be discussed in a later section, for Florensky this one Truth was anticipated not only in Trinitarian theology, but also in the Absolute Infinity at which Cantor had arrived at the end of his reflections on actual infinity; which both Florensky and Cantor identified with God; and which could never be arrived at through the smooth continuity of a potential infinity.

But how, through what kind of discontinuity, are we then to approach this Absolute Truth that Florensky identifies as the highest mode of Absolute Infinity? Florensky proposes a preliminary answer in his Second Letter, called simply "Doubt." He begins with the foundational thought of modernity, discovered by Descartes, that "for theoretical thought" the one Truth, "the Pillar and Ground of Truth," is *certitude*. And Florensky analyses the attempts made by the soul hungry for Truth to fulfill this demand for certitude, first through various modes of givenness, which never lead beyond the self-assertion of I=I and A=A discussed already, and secondly by an analysis of the futile attempt of rationality or discursive thought – the endless pursuit of one explanation after another – to arrive at anything more than yet one more truth, which leads to an endless sequence of successive truths, where every A is derived from a not-A, which must in turn be derived from what is not-not-A, and so on. Modern thought, then, leaves us with the choice between

> an impenetrable wall and an uncrossable sea, the deadliness of stagnation [in the A=A] and the vanity of unceasing motion [in the endless regression from A to its explanation by not–A]; the obtuseness of the golden calf and the eternal incompletion of the Tower of Babel (*Pillar*, pp. 26f).

In a subtle and complex dialectic that cannot be easily summarized, Florensky proceeds through skepticism and probabilism to a final impasse, in which the *longing* for the Truth, whose light manages to penetrate the darkness of the I=I, lures the seeker to a willingness to go beyond this bubble of self-identity, not just in an endless quest for yet another conceptual not-A, which will in turn become subordinated back into the circle of self-identity, but to leave the sphere of the I altogether – to break with self-identity in a radically discontinuous movement that is nothing less than, for the I, a death unto itself and to the "lower law of identity," in order to be reborn through the achieving of an impossible identity with what is not-I, discovering in the process a higher, truer law of identity, a "spiritual law of identity" (*Pillar*, p. 348). If Parmenides' "untrembling Heart of immutable Truth," and with it the ontological ground of the law of identity, is to be reached, then the path must lead not through the serene, ethereal heights into which daimonic charioteers had carried the Eleatic, but through the Garden of Gethsemane (*Pillar*, p. 45).

Consonant with all the great traditions of spirituality, then, Florensky argues that it is only through a kind of intellectual *ascesis* – like the casting-off of all that is cumbersome to the athlete in training, as the word once suggested for the ancient Greeks – that the highest truth can be found. The image upon which Florensky draws here is Abraham, the father of faith, and the father of peoples, who is called to leave behind his ancestral home for an unknown land, a new land, a "better" and indeed "divine" country (*Pillar*, p. 55; *Heb*. 11:8, 14–15). Likewise, the knower must leave behind his own self-identity, leave behind the law of identity itself, cross over the abyss of rationality and go out to another – another who cannot be proved, because He is Himself a "self-proving Subject," which alone could be Absolute Truth (*Pillar*, pp. 33ff). Moreover, this "going out" must at the same time be an "entering in," an ontological union with the Truth who alone can be considered as "*actual infinity*, the Infinite conceived as integral Unity, as one Subject complete in itself" (*Pillar*, p. 33). Thus,

> the act of knowing is not only a gnoseological act but also an ontological act, not only ideal but real. Knowing is a real *going* of the knower *out* of himself, or (what is the same thing) a real going of what is known into the knower, a real unification of the knower and what is known (*Pillar*, p. 55).

But this is to say that knowing is itself a mode of love: "in love and only in love is real knowledge of the Truth conceivable" (*Pillar*, p. 56). "Love takes the monad out of itself" and "unity in love is that which takes each monad out of the state of pure potentiality, i.e. spiritual sleep, spiritual emptiness, and amorphous chaos" (*Pillar*, p. 236). And in knowing (and loving) Absolute Truth, which is related to every individual truth as actual infinity is to every finite element – as including them, without being contained by them – it is possible to then know (and love) finite things as well, within that Absolute.

> Knowing is not the capturing of a dead object by a predatory subject of knowledge, but a living moral communion of persons, each serving for each as both object and subject. Strictly speaking, only a person is known and only by a person (*Pillar*, pp. 55f).

Hence, in a manner entirely different from the way it is argued by Spinoza, every truth known is a truth known about God. But, rather than God being dissolved into the world, the world is itself personalized within the God whose very energies it manifests, yet who nevertheless essentially transcends it. "God is transcendental for the world, but the world is not transcendental for God: rather it is wholly permeated with divine energies" (*Pillar*, p. 363). Thus, this mode of knowing that frees the self from its own self-imprisonment, that is itself a mode of love allowing every truth to entail a personal relation to God, is possible only because in each case the initiative always already proceeds from God. "God's love goes over to us," and indeed, it is this divine love itself that has lured the self beyond itself, enticed the I to find itself in unity with the not-I (*Pillar*, pp. 56f).

Happily, however, we need not somehow plunge into mystical unity with God all at once, and with no preparation. There are certain modes of knowing within which we are offered an anticipation, a preparation – nothing less than a "preliminary hint... of the heavenly in the earthly":

> This revelation occurs in the personal, sincere love of two, in friendship, when to the loving one is given – in a preliminary way, without *ascesis* – the power to overcome his self-identity, to remove the boundaries of his I, to transcend himself, and to acquire his own I in the I of another, a Friend. Friendship, as the mysterious birth of *Thou*, is the environment in which the revelation of the Truth begins (*Pillar*, p. 283).

Crossing the abyss, making the leap, entering into this radical discontinuity, going from the life that is a kind of death, the empty self-identity of the I=I, into a death (I= not-I) that is a kind of life, the soul discovers a "new" self, finds the truth that only by losing oneself can ones self be found. But once again, we need not think that this discontinuous exit from the monadic hegemony of self-identity necessarily requires some dark night of the soul, an anguished state of mystical longing such as we find in some of the Western mystics. It can take place, to some degree, in the moment when some wisp of cloud, or an ancient scent lingering in the autumn air, or the song of a mockingbird in the calm depths of a Southern night, penetrates our shell and moves us beyond and outside ourselves, i.e. the moment in which we, however briefly, embody "the act by means of which a creature is liberated from its selfhood and goes out of itself" (*Pillar*, p. 235). We are made, Florensky argues in harmony with Patristic Christianity, in the image of God. And thus, remarkably,

> to love visible creatures is to allow the received Divine energy to reveal itself – through the receiver, outside and around the receiver – in the same way that it acts in the Trihypostatic Divinity itself. It is to allow this energy to go over to another, to a brother (*Pillar*, p. 62).

6. The Uses of Contradiction

It would be a mistake, however, to see Florensky's critique of the law of identity as a form of irrationalism, similar either to that of Breton and Duchamp in France, or to that articulated by his Russian contemporary Lev Shestov and his admirer, D.H. Lawrence. Florensky was first of all a mathematician and scientist, and long after his philosophical voice was silenced, he continued work in these fields. Rather, Florensky is appealing to a distinction and contrast that goes back to ancient Greek philosophy – and which was important to his Slavophile predecessors such as Khomyakov –

between lower and a higher modes of knowing, between *dianoia* and *nous*, between what Florensky terms in Russian *rassudok* or "rationality" and *razum* or "reason," or between discursive rationality, which seeks to explain conceptually, and what German Idealism called intellectual intuition, which grasps higher truths through non-sensuous immediacy (*Pilllar*, p. 7). While the former is fragmented and divisive, the latter is integral and unifying, drawing people together into a kind of loving concord that in Russian is called *sobornost* (*Pillar*, p. 430). And while "rationality" insists upon the "lower" law of identity, "reason" transcends it and operates according to a "higher," spiritual law of identity.

Kant, of course, argued that such intellectual intuition was impossible for human beings, and employed a series of antinomies, or equally compelling arguments supporting contradictory conclusions, which arise when human understanding tries to go beyond the limits of empirical experience. Yet something on the order of *nous* or *theoria* or *contemplatio* (or intellectual intuition) has until modernity been seen by philosophers as the highest mode of knowing, from Parmenides to the Middle Ages. In his retrieval of *noesis* through *ascesis* and the experience of religious mystery, Florensky shows just how deeply the roots of patristic epistemology extend into ancient Greek philosophy, which characteristically (and notably in Parmenides, Plato, Aristotle, and Plotinus) saw noetic rationality as the fulfillment of the human condition, the mode in which (however they articulated it) human beings could come closest to the divine – yet one that for the ancient philosophers, for the most part merely flickered on the horizon, reachable if at all only for a few, and only then for brief periods. And indeed, modernity itself may be defined by its very rejection of noetic or contemplative knowledge, this purportedly direct or immediate apprehension or intuition of higher, eternal, transcendent realities, which traditional, patristic Christianity saw as the birthright of all the faithful who undertook the *ascesis* of the *ekklesia*, the ancient Christian community. Florensky, then, may be seen as undertaking the most significant attempt to justify this putatively higher rationality since the German Idealists had sought to overcome Kant's limitations on human knowledge. But just as Feuerbach and Marx saw the need to go beyond German Idealism not within theory, but through an exodus from theory into praxis, so too (in a very different mode) Florensky also seeks to justify higher knowledge through something active and engaging – through experience, and through the love that takes the knower beyond the bounds of self-identity and the law of identity itself, i.e. through an ontological migration from self-identity to identity with the other.

How to activate or engender this higher mode of reason? In Book VII of the Republic, Plato had posed the question of what "*would be apt to summon or stimulate noetic activity*" (523e; [9], p. 202, translation altered). And Socrates here engages his interlocutor Glaucon with a strange exercise, asking him to hold up his fourth (ring) and fifth (little) finger, and report whether the fourth finger is little or small, to which he answers that it is large. Next, he asks Glaucon to hold up his third (middle) and fourth fingers, upon which Glaucon reports that the same finger, the fourth, has now become little. The same thing, the fourth finger, is thus both itself and not itself, both big and small. And this contradiction in the visible realm – and this encounter with what Plato in his later philosophy called the indeterminate dyad – is precisely what he maintains is able to stimulate and awaken the noetic intellect to go beyond the visible toward what is intelligible, but not visible: to make the transition from one world to another. Likewise, Florensky takes the concept of antinomy, which to Kant was a warning sign beyond which we must not advance, as in fact a spur to awaken our noetic powers.

"Rationality," clinging to the illusory safety of the I=I and the law of identity, must undergo the discipline of *ascesis*: the rationalistic mind must be "tamed," i.e. it must forgo its own pretensions to absoluteness, in order to arrive at a genuine Absolute (*Pillar*, pp. 7, 23). And it is precisely the great antinomies or mysteries of religion upon which this discipline and taming must be exercised:

> The mysteries of religion are not secrets that one must not reveal. They are not the passwords of conspirators, but inexpressible, unutterable, indescribable experiences,

which cannot be put into words except in the form of contradictions, which are 'yes' and 'no' at the same time (*Pillar*, p. 117).

Thus, when these mysteries of religious experience are put into words, they become antinomies embracing both thesis and antithesis – or to use a word that for Florensky is synonymous with religious antinomy, they become *dogmas*.

The basis of dogma would thus be not some kind of mandate based on "blind faith," but quite the opposite: dogma would in this case be the "mind's eye," or rather "that eye by which mankind looks at the inaccessible light of ineffable Divine glory," but stated in conceptual language (*Pillar*, p. 79). Moreover, we should expect *beforehand* that whenever these mysteries and this noetic experience are translated into conceptual language, the discourse of rationality, the result will be manifest as an antinomy. Moreover, it is just this antinomic character that should stimulate rationality to purify and discipline itself, in order to arrive at "living religious experience as the sole legitimate way to gain knowledge of the dogmas" (*Pillar*, p. 5). The usual proofs for the existence of God and all the other attempts to create what Florensky sees as the absurdity of a "rational faith" would thus be proceeding in precisely the wrong direction. "So-called 'rational faith,' faith with rational proofs... is a harsh, cruel stony growth in the heart, which keeps the heart from God." Rather, "the truth is known only through itself" (*Pillar*, p. 48). Even the very "existence of Truth" is "not deducible but only demonstrable in experience.

What are examples of such dogmas that invite the soul to proceed beyond the safety of its own self-identity? Surely, and above all, we must list the dogma of the Self-proving Subject, the Trihypostatic Unity which through the unity of its own embrace of otherness with itself, invites us into the very loving dynamic which has been the ontological mode of God from eternity. But there are more accessible examples, and Florensky cites many of them in *The Pillar and Ground of the Truth*. There is, for example, what he calls the antinomy of *philia* and *agape*, that salvation is esoteric and for the elect, *and* that it is open to everyone. Or that one should "preach the gospel to every creature" (*Mk* 16:15) *while at the same time* "neither cast ye your pearls before swine" (*Mt* 7:6; *Pillar*, pp. 300f; see also pp. 295f). Or there is the great antinomy of faith and works, i.e. "between God's grace and human *ascesis*" (*Pillar*, p. 255). Indeed, sometimes the antinomy is presented in a single passage (*Phil*. 2:12–2:13) of scripture: "Work out your own salvation with fear and trembling" (the thesis) "for it is God which worketh in you both to will and to do of his pleasure" (the antithesis). Or the antinomy may reveal itself within a few pages of a single Gospel: "For judgment I come into the world" (*John* 9:39) and "I come not to judge the world" (*John* 12: 47). Thesis and antithesis must both be embraced simultaneously, not through conceptual explanation, but through rising to the kind of noetic experience to which these binary realities in each case point.

Again, Florensky's affinity for paradox, first honed in his work with the paradoxes of infinity around which so much of Cantor's work revolves, is pivotal in his theological and philosophical insights here. One of the most important appendices of *The Pillar and Ground of the Truth* discusses how the problem of irrational numbers, long dismissed as "fictitious numbers" and "*numeri surdi*," propel us to break through and leave behind the "circle of operations which arithmetic knows... in order to be born into a new, hitherto unseen and unthought of world" (*Pillar*, p. 362). This is, he argues, the world of actual infinity, entered through the portals of the paradoxes generated by the juxtaposition of the finite and the infinite, effecting "a leap, a discontinuity in development." These insights into the role of paradox, contradiction, and antinomy cast new light upon Christ's use of *parables* in his teaching, which usually entail an antinomy, a set of opposing insights that must both be embraced. They allow Florensky important insights into the relation between these two modes of rationality themselves, while aligning him against the one-dimensional rationality of modernity, and alongside traditional religious discourse, such as is common not only in the enigmatic paradoxes of Taoism, Zen Buddhism, Hinduism, and Sufism, but above all in the splendid paradoxes evoked by so many of the great Orthodox *Kontakia* and *Stichera*, especially those celebrating its holiest feast days, each of which centers upon a paradox: "Thou hast dwelt in a cave, and hast lain down in a manger, O thou whose throne is in heaven... The Unseen is seen, the Untouchable is touched, the Beginningless beginneth"; "He who hung the earth upon the waters is hung upon the Cross... He who wraps the heaven in clouds is wrapped in the purple of mockery" ([2], pp. 411f; [8], p. 609).

But when it encounters this antithetical character of dogma, being foreign to the experience that engenders and underlies it, "the rational mind involuntarily shudders," for it senses "that it is required to sacrifice itself" (*Pillar*, p. 121). Rationality does not have the taste or the capacity to bring together thesis and antithesis, for "only religious experience apprehends antinomies and sees how their reconciliation is possible" (*Pillar*, p. 120). Rather, in its refusal to go beyond the security of its own self-identity, rationality clings to one side or another of the religious antithesis –

> a one-sided proposition takes the place of absolute Truth, and such a proposition thus excludes everything in which is seen the antinomic complement to the given half of the antinomy, rationally incomprehensible.

The Greek word for choice is *airesis*, which came to mean "one-sidedness," and which forms the root of the English word "heresy." Thus, choosing one side or the other, thesis or antithesis, this one-sidedness of rationality is necessarily sectarian, "heretical," or one-sided: "a heresy, even a mystical one, is a rational one-sidedness that claims to be everything" (*Pillar*, p. 119).

7. From Actual Infinity to Absolute Infinity

Surely the greatest paradox discovered by Cantor, and doubtless the one that meant the most to him, as it did to Florensky after him, was that there were higher and lower orders of infinity, leading up to an absolute infinity that exceeds comprehension altogether, and that both men identified with God. Once Cantor began to take seriously the concept of actual infinity, as opposed to the merely potential infinity familiar from the paradoxes of Zeno and the ordinary concept of endless iteration, the paradoxical notion of a hierarchy of infinities began to pose itself – paradoxical, because it would seem that infinity is something that cannot be exceeded. And yet he came to understand that there was, for example, a lower infinity of the integers, and then a higher infinity of the integers plus all the rational and algebraic numbers. Beyond this was a yet higher infinity of what he called transfinite numbers, those irrational numbers (such as "Pi") that were not algebraic (i.e. capable of being designated by a formula, such as "the square root of two"), and whose infinite number so far exceeded all the preceding infinite sets taken together that the ratio had to be rounded to 1 – i.e. if the rational and algebraic numbers were mixed together with the transcendental numbers, the probability of randomly choosing a transcendental number would be one, and the probability of choosing one of the infinite number of integers, or one of the infinite number of rational fractions, or one of the infinite number of algebraic numbers would be zero ([1], pp. 90, 132)! And of course, the movement from a lower infinity to a higher one is necessarily *discontinuous*.

Yet the infinity of the transcendental numbers still did not stand at the top of the hierarchy. For Cantor, to whom Florensky refers to as "the founder of the modern theory of actual infinity," the realization that there were a hierarchy of infinities – at the pinnacle of which was what he variously understood as the "set of all sets," or "the totality of everything conceivable" – led him to an absolute limit to mathematical understanding, something that "cannot be known, not even approximately," and which he called absolute infinity, or simply "the Absolute," and sometimes compared to the "One" of Plotinus (*Pillar*, p. 574; [6], pp. 55, 95). Thus, for Cantor, this Absolute was by no means an abstraction, but rather that which was most real of all:

> it is the single, completely individual unity in which everything is included, which includes the Absolute, incomprehensible to the human understanding. This is the *Actus Purissimus*, which by many is called God.[7]

Perhaps Cantor was himself a mystic, having not merely arrived mathematically at the incomprehensible concept of this Absolute that he called "God," but had in some sense encountered this reality in experience. But it is clear that Florensky's main innovations beyond Cantor were (a) to show a path not just theoretically, but within religious experience to this Absolute, and (b) to show that this Absolute Infinity could only be grasped by employing the higher rationality discussed above, as Trihypostatic Unity and Self-Proving Subject, i.e. as the Trinitarian God of Patristic Christianity and Orthodox Faith. It is one of Florensky's main theses that there are ultimately only two choices: *either* the endless futility and hopeless despair of the "bad infinity," i.e. the *potential infinity* that ceaselessly seeks what it can never have – the central dynamic of torment to many of the figures in Dante's Inferno – *or* ecstatic fulfillment of the search for Truth in the actual infinite, ecstatic because it entails a "going beyond" itself for rationality and self-identity, or in theological terms, a *kenosis* or self-emptying, the sacrificing or abandonment of oneself that makes possible a new, and higher kind of existence. But is there an actual infinity, let alone a hierarchy of actual infinities? And if so, what character would this highest order of infinity possess? Finally, through what path could experience arrive at the highest level of actual infinity?

Florensky's answer to these three questions is extraordinarily rich and complex, and can only be addressed in outline here, although it should be possible to at least sketch out an answer to them, for they will help illumine the other main topics of this paper (the law of identity, discontinuity, and antinomy). First, Florensky makes some very simple observations concerning what he regards as "the fundamental and wholly elementary distinction between *actual* and *potential infinity*," a distinction that he feels has recently suffered from error and neglect (*Pillar*, p. 351). Both potential and actual infinity are quanta, like any other kinds of quantum. But potential infinity is a *variable quantum*, changing in relation to any other quantum with which it may be compared, since by definition it must exceed any given quantum. Thus, potential infinity is not a specific quantum at all, but simply "a special way of considering a quantum," i.e. that it is indefinitely variable. Thus, potential infinity is not something actual at all, but an *ens rationis*, an entity posited by rationality. Its infinite character never actually exists, but is always variable, in process, underway, and thus it is never fully itself. It is what the ancient Greeks called the *apeiron* and viewed disparagingly, and what German Idealism called *schlechte Unendlichkeit,* "bad infinity," the infinity of the ceaseless "etcetera." And as we have seen already, Florensky associated this with the endlessness of desire and dissatisfaction, of unsatisfied striving, of a movement that can never achieve its goal and for which it is impossible to ever find peace – for as soon as it varies to exceed one quantum, there remain endless greater quanta which it must still exceed (*Pillar*, pp. 351f).

Actual infinity, in contrast, is complete in itself, and thus is not a variable quantum at all, but a *constant quantum.* It is always already fulfilled, fully itself. As a simple example, we may take the set of all points inside a certain closed figure, such as a circle or square. Since the figure is bounded, the number or points within it is complete and constant, fully determinate, rather than variable. Yet it is at the same time infinite, since the number of points exceeds each of the numbers in the series 1, 2, 3, ..., *n* ... and is greater than them. It is, then, an actual infinite. Or, to give a more theologically significant example,

> we can say that the powerfulness of God is actually infinite, because it, being determinate (in God there is no change), at the same time is greater than all finite powerfulness (*Pillar*, p. 353).

Moreover, Florensky adds, the concept of actual infinity is more basic than that of potential infinity. For in order for potential infinity to be possible, there must be an already infinite domain within which its ceaseless variations can endlessly proceed. That is, "*every potential infinity already presupposes the existence of an actual infinity as its super-finite limit*" (*Pillar*, p. 353, italics in original). Moreover, it is also important to observe that no actual infinity can be gradually reached through the variation process of potential infinity, for between actual infinity and the infinite increase of a quantum that we consider potentially infinite, there is a radical *discontinuity* –

not necessarily unreachable, but certainly not attainable through increase along a progressive continuum, which could only aspire "farther and farther, without ever being able to achieve a synthesis and to find peace in the whole" (*Pillar*, p. 353).

Now we may return to the *aporia* discussed earlier between givenness and discursion, neither of which alone could provide a successful path toward Truth, leaving us with the dilemma of choosing between the lifeless desert of the here and now, which intuition offers, and the torment and bad infinity of endless explanation, which never arrives at its goal – between the egoistic assertion of a particular givenness, certain merely because it is *my* givenness, and the ceaseless discursivity that continually seeks to explain every A by some new not-A, i.e. between the law of identity and the law of sufficient reason. Yet Absolute Truth would somehow need to possess both characters. On the one hand, if it is to be experienced, it must be *given* in experience, arrived at by *finite intuition*. But if it is to be more than arbitrarily asserted, it must be exhaustively explained, and the grounds for it as a judgment absolutely proved, and this could only be possible not through a potentially infinite process, but within the actual infinity of an already completed *infinite discursion*. Absolute Truth, then, would need to be both finite infinity and infinite finitude, both actually infinite in having already synthesized its grounds, and at the same time capable of being intuited as a given, i.e. it must be a "unity of opposites, *coincidentia opositorum*" (*Pillar*, p. 33). Moreover, since finite discursion cannot itself provide for it the actually infinite synthesis of all its grounds, Absolute Truth would have to be self-proving or self-grounding, a feature that we saw earlier (in the discussion of numerical identity) is characteristic only of a person or *subject*. Absolute Truth, then, if it exists, would be our experience of an Absolute Self-Proving Subject. And we have seen already how the *kenosis* that leads beyond self-identity and the *ascesis* that leads beyond rationality open the self for the experience of such a Self-Proving Subject. But is there such a reality? Florensky is clear that this must be discerned through ascetic experience alone: the Truth cannot be known beforehand, nor can it even be known for sure whether it exists, but rather it must be encountered in experience. He is able to show only that there must be such a Self-Proving Subject if there is to be not just truth, but the Truth; for in the same way that actual infinity provides the domain for potential infinity, Truth would itself be necessary even for a single finite truth to be possible. Thus, Florensky concludes, "rationality is possible not in itself but through the object of its thought, and if, and only if, it has an object of thought in which both contradictory laws of its activity, i.e. the law of identity and the law of sufficient reason, coincide." And in addition, we must add that

> rationality is possible if Absolute Actual Infinity is given to it. But what is this Infinity? It turns out that such an Object of thought, making thought possible, is the Trihypostatic Unity.[8]

But we may carry this yet another step further. To be fully a subject, such an Absolute subject would have to go beyond itself, to enter in love into another: "this Subject is such that it is A and not-A" (*Pillar*, p. 36). Let us, then, designate this not-A as B. But what is B? B too must go out from itself, transcend itself, in order to be a personal reality. But if B is merely not-A, then its going over in love to not-not-A would end up with the result that A has never really left itself at all, i.e. with A returning to itself. For if A=B, and if B=A, then we have not left the solipsistic self-identity of A=A. Thus, B must be something more than not-A, which we can designate as C. But here, Florensky concludes,

> through C the circle can be closed, for in its 'other,' in [B understood as] not-C, A finds itself as A. In B ceasing to be A, [i.e. through B finding a not-B which is not simply A] A receives itself mediately from another, but not through the one with which it is equated, i.e., [it receives itself] from C. And here it receives itself as already 'proved,' already established. The same thing goes for each of the subjects A, B, C of the triple relationship (*Pillar*, p. 36).

Or, as he summarizes, "Truth is the contemplation of Oneself through Another in a Third: Father, Son, and Spirit."[9] But this contemplation is far from being a lifeless, bloodless "theoretical" state. Rather, "Absolute Truth is known in love" – not in love as a psychological condition but love as a metaphysical act, the love that makes possible the leap beyond the bad infinity of self-identity into the actual infinity of ontological communion (*Pillar,* p. 67). Thus, Florensky has shown a path whereby experience itself can "go beyond rationality, to enter the domain where rationality with all its norms is rooted" (*Pillar,* p. 44).

The First Nicene Council, which established the initial and guiding understanding of Christian thought, can be seen as primarily the search for the right word, a word that Florensky takes as central for his entire mathematical-philosophical-theological project. For the Greek word upon which the great Nicene Council of 318 finally settled as its cornerstone – the foundational word of Patristic Christianity and the fundamental word for this identity of substance (*ousia*) that is constitutive of personhood – is *homoousios*, "of one substance," or "consubstantial." It is for Florenksy the true Principle of Identity, not the impoverished and paranoid self-identity of I=I, but the fulfilled, peaceful identity between Same and Other by way of a Third. If he is right, it is the great, foundational principle of ontology. "It is impossible," exclaims Florensky here,

> to mention without reverent fear and holy trepidation that moment – infinitely significant and unique in its philosophical and dogmatic importance – when the thunder of *Homoousios* first roared over the City of Victory [i.e. Ancient Nikea, City of *Nike*] (*Pillar,* p. 41).

Thus, Patristic Christianity can be seen, and indeed was seen by many of the Church Fathers (such as the Alexandrians and the Cappadocians) who were well versed in Greek philosophy, as offering the solution to what is arguably the great unsolved philosophical problem of antiquity: as articulated in Plato's *Sophist*, it is the problem resolving the unstable relationship between the Same and the Other, without ending up in the state of perpetual warfare entailed by dualism, or the state of inescapable totality entailed by monism. Moreover, it demonstrates the ontological identity between thought and being that was sought and posited by both Heraclitus and Parmenides, through the experience of the Love of the Persons of the Trinity for one another, by means of the grace-given identity with the very dynamic of that love itself. And this would, at the same time, be the experience of the identity between Reason and Truth, between thought and being, between God and humanity, between the world of fluttering leaves and the Center toward which they, along with all things, are drawn. As Florensky writes in his concluding paragraph, "The Triune Truth Itself does for us what for us is impossible. The Trihypostatic Truth Itself draws us to Itself" (*Pillar*, p. 348).

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Notes

1. Pavel Florensky, *For My Children*, trans. in [7], p. 9. Pyman has an illuminating discussion of Florensky's early, untranslated essay, "On the Empirical and the Empyrean" on pp. 41–45. Written in 1904, it provides an important prolegomenon to his major work, *The Pillar and Ground of the Truth*, much of which was written by 1908, but which was not published until 1914.
2. Ibid., emphasis added.
3. Ibid.
4. "In contrast [to Kant] I was always a Platonist... the appearance was for me always the appearance of the spiritual world" And thus, "the appearance – two-in-one, spiritual-material symbol – was always precious to me in its immediacy." Florensky, *For My Children*, my own English translation from the German translation [5], p. 212.
5. S. S. Demidov, "*O matematike v tvorchestve P.A. Florenskogo*," p. 171, cited in [7], p. 260.
6. It would not be wrong to see this realism as something anticipated in the anthropologies of Aristotle and Thomas Aquinas, who both argued that in a very limited sense, the knower and known become united: what is known, in the very act of being known, assumes a new being in the understanding of the knower. But Florensky here is proposing something much more radical – more radical even than Hegel, who understands the self as needing to discover itself through its relation to the other. Rather, for Florensky, the knowing self must unite in love with the known, both in order for meaningful knowledge to take place, and for the self to be a concrete self. It is, one might say, an *erotics* of identity.
7. From one of Cantor's last letters to the English mathematician Grace Chisholm Young, cited in [1], p. 189.
8. *Pillar*, p. 347. For Florensky's discussion of the *Absolutum* in Georg Cantor's work, see *Pillar*, p. 354.
9. *Pillar*, p. 37. Florensky offers a detailed comparison between the Christian understanding of Trinity and the views found in non-Christian religions, such as the Hindu triad of Brahma, Vishnu, and Shiva, on pp. 478–482.

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On Contraction in Orthodox Philosophy

Ni la contradiction n'est marquee de fausseté,
ni l'incontradiction n'est marquee de vérité.
Blaise Pascal (Penseés, p. 384)

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Abstract:
Orthodoxy sometimes appears to lack a respectable form of logical reasoning. This is because objective mystery is so central, and because contradiction is, therefore, a methodological necessity. However, the belief system also rejects explosion. Thus, on the one hand, the law of non-contradiction is violated, and, on the other hand, it is respected. In terms of thinking about Orthodox thinking, this is the fundamental issue, namely that logical reasoning in Orthodoxy is paraconsistent. That is what we examine in this essay.

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As with other religious logical systems,[1] we find that in Orthodox Christian thought there are instances of tolerated inconsistencies. Moreover, the principle that anything follows from a contradiction, ex contradictione quodlibet (henceforth ECQ), is not embraced. To embrace ECQ is to maintain that given any proposition of the form p ~p, which, by the inference rule of simplification, means that p is a premise and ~p is a premise, then any q may be inferred by the addition rule such that p ∨ q; and this inferential process can continue ad infinitum, thus implying the truth of any and all sentences. That is, {A, ~A} |= B (the so called inference of explosion), where B is the variable for quodlibet, is taken as a valid consequence relation. For writers in the Orthodox tradition, failure to maintain ECQ is implicit. This is largely because Orthodox tradition stretches back some 2,000 years, whereas ECQ has become widely embraced only in about the past 150 years. Nevertheless, whether or not one is aware of the apparent problematic logical implications, the Orthodox belief system includes two basic contradictions; and insofar as they are affirmed in isolation (that is, insofar as these but not all inconsistencies are tolerated), then ECQ is tacitly invalidated. Long before the development of modern logics, and the widespread endorsement of ECQ, a basic assumption is present and pervasive in Orthodox thought—namely that Christian belief is coherent even if it is not consistent. That assumption led thinkers like Athanasius and the Cappadocian Fathers to reject inferences in keeping with the law of non-contradiction (henceforth LNC)—but not necessarily the LNC itself—and to promote a (tacit) paraconsistent inferential methodology. This means that Orthodox Logic is therefore paraconsistent. For the Orthodox theories of God and Jesus Christ are inconsistent but not incoherent and explosive.

The two contradictions endemic to Orthodox thought are that God is both one and three and that Jesus is both (fully) God and (fully) man. From the perspective of non-paraconsistent logics, these propositions are in fact contradictory; so they entail the truth of any and all sentences. These propositions are either contradictory and, thus, untrue, or they are contradictory, yet true. The logic of Orthodox thought, of course, affirms the latter. Our examination of this is twofold. First, we define the term contradiction, and briefly explicate the LNC. Secondly, we turn to Pavel Florensky (1882–1937), who was the first Orthodox philosopher to understand both the form of logical reasoning endemic to Orthodox thought as well as its paraconsistent implications.

It has been maintained by Ayda Ignez Arruda, and widely held since, that the earliest development of paraconsistent logics occurred in Russia in or around 1910 by Nikolai Alexandrovich Vasiliev (1880–1940).[2] A similar trend was also taking shape in the work of Jan Łukasiewicz (1878–1956) in Poland, see [22], [23]. However, as already indicated, paraconsistent logic has long been an implicit feature of Orthodox thought. Several writers even deny the LNC (though most hold, at least implicitly, to both LNC and Orthodox dogma without recognizing that such a position is itself inconsistent), yet it is not until Florensky that we have an attempt to justify allowance of inconsistency in Orthodox thought. Thus, in terms of making explicit what had long been implicit in Orthodox thought, Florensky has played a role in the development of paraconsistent logics. His ability to do so more extensively was almost undoubtedly inhibited by the Revolution. But that he has not been seen to have a role in developing paraconsistent logic is understandable for at least two reasons. First, like the vast majority of his very large bibliography, his magnum opus has been accessible to scholars (until quite recently) only in Russian. Secondly, his forays into paraconsistent logic come to us, in that text, not as a treatise on logic, but rather as part of a work that, as he puts it, is 'for Catechumens' [13], p. 6 (hereinafter PGT). It was first published in Moscow in 1914 as Столп и Утверждение Истины: Опыт Православной Теодицеи в Двенадцати Письмах. That publication date puts his ideas in the developmental stages of paraconsistent logic. It is possible, of course, that Florensky had read or been otherwise exposed to the ideas of Vasiliev. And since he studied mathematics at Moscow University with Nikolai Vasilievich Bugaev (1837–1903), Sergei Nikolaevich Trubetskoy (1862–1905) and Leo Mikhailovich Lopatin (1855–1920), it is perhaps even probable that there may have been some influence. But from the evidence available in the PGT, we are compelled by charity to conclude that there was no such influence. Of course, it may be the case that there was, but that Florensky just did not cite Vasiliev when he should have. That is indeed possible. But Florensky is quite fastidious in citing his sources. So much so that the notes in the PGT run some 160 pages in what appears to be 9 pt font (not to mention his 'Clarification and Proof' section, roughly 75 pages). Thus, it is more improbable than probable, in our view, that Florensky was influenced in any way by Vasiliev. It is more likely that he was influenced by the neo-Kantian thought of Alexander Ivanovich Vvedensky (1856–1925) and Ivan Ivanovich Lapshin (1870–1952), both of whom speak of violating the law of non-contradiction and were less neglected than Vasiliev.[3] But there is no indication in the PGT of that either. Thus, Florensky's ideas on paraconsistency are almost certainly original. That he appears to have been among the first to attempt to develop a paraconsistent logic is important as much for logic as for Orthodox thought. Before saying more about Florensky, though, we must frame the discussion a bit.

What we need initially is an answer to the question 'What is a Contradiction?' That can only be had with a definition of the term. The English term itself derives from the Latin verb contradictio (contradicere), 'I speak against' ('to speak against'). But the initial definition of 'contradiction' comes to us from Aristotle. In the Greek the term Aristotle used was antiphasis. That term is composed of two Greek words. The term anti is a preposition. In this use, it means 'against.' The second term, phasis, comes from the verb phēmi, which means 'to say, speak or tell.' It connotes the act of expressing opinion, thought or belief, and, thus, of having an opinion, thought or belief. The term phasis itself means a 'saying, speech, sentence, affirmation or assertion.' A fair etymological definition of the term antiphasis, then, is that it means a 'saying, speech, sentence, affirmation or assertion against.' So Latin and Greek provide the same basic meaning. But both leave us with the question against what? And we shall answer that in due course. For now, however, we need to look at Aristotle's own definition of the term.

Actually, we should probably say definitions. For, in addition to defining it a time or two in the Organon (Cat. 10.13b, 28ff.; de Int. 6.28–37; 7.17b, 38–18a, 7), Aristotle also defines antiphasis twice in the Metaphysics (cf. 1005b, 13–22 and 1011b, 13–14[4]). We turn first to the definition given in de Interpretatione. For, in addition to incorporating the term antiphasis, which does not appear in the Categories passages, the definition we get in de Interpretation uses termini technici, which become important for Orthodox thought. (This is very explicit, as we shall see below, in John of Damascus.) Those terms do appear in Categories. And although they themselves have not been equally as influential in logic or philosophy, especially Analytic thought (we do see some use in Continental philosophy though, notably, e.g., in Jean-Luc Marion's L'idole et la distance [Paris: Editions Bernard Grasset, 1977][5]), their respective concepts have been. But what terms do we have in mind? First, Aristotle speaks of a true statement, an affirmation, as kataphasis apophansis, a 'positive proposition.' Second, he speaks of a false statement, a denial, as an apophasis apophansis, a 'negative proposition.' The two terms kataphasis and apophasis, then, are what we have in mind. In most cases, Aristotle is not the direct source of those terms for Orthodox thought. For a philosopher such as Pseudo-Dionysius (c. 500), this bit of Aristotelian logic comes from late ancient Neoplatonic thought, particularly Proclus' Elements of Theology, where he gets much of his Aristotelian influence; but most Eastern patristic writers rely on the Isagoge for their knowledge of Aristotle's logic. These words bear a similarity to antiphasis. Both are composed of a preposition plus phasis. The two prepositions in question are kata and apo. The first of these, in this context, takes the meaning of 'according to' or 'in agreement with.' Thus, etymologically kataphasis probably means something like 'according to/in agreement with saying, speech, sentence, affirmation or assertion.' That's a bit wooden. A better rendering is 'according to/in agreement with expression.' The second preposition, apo, implies the idea of being 'away from,' 'at a distance from' or 'far from.' The term apophasis, from an etymological perspective, denotes the idea of being 'away from/at a distance from/far from expression.' These terms distinguish between two types of propositions: kataphatic and apophatic propositions. Aristotle affirms that pasē kataphasei estin apophasis antikeimenē kai pasē apophasei kataphasi, 'every kataphasis has an opposite apophasis, and similarly every apophasis an opposite kataphasis' (de Int. 6, 33–5[6]). This is what he calls an antiphasis. As (existential and universal) examples, he gives 'Socrates is white' (p) and 'Socrates is not white' (~p), and 'every man is white' and 'not every man is white' (de Int. 7.18a, 1–2). In Categories, Aristotle argues that this distinction (as opposed to contraries, correlatives, positives and privatives) always involves truth and falsity. Thus, it is either the case, for example, that 'Socrates is ill' or that 'Socrates is not ill' (Cat. 10.13b, 28ff). Even in theory, it cannot be the case that 'Socrates is both ill and not ill.' John of Damascus (c. 650–ante 754[7]) picks up on this definition in his Philosophical Chapters[8] (ch. 63). His examples of kataphatic propositions are 'Socrates is wise' and 'Socrates walks.' For apophatic propositions, he gives 'so-and-so is not wise' and 'so-and-so does not walk' (cf. [7], p. 97; for the Greek I have used PG 94). A contradiction, or antiphasis (John also wrote in Greek), then, is understood in terms of opposition between kataphatic and apophatic propositions. Likewise, John follows Aristotle (via Ammonius in Cat.) on kataphasis and apophasis. He defines kataphasis as 'the stating of what belongs to something, as, for example, 'he is noble.'' And apophasis is 'the stating of what does not belong to something, as, for example, 'he is not noble' [7, p. 88]. Thus, for John, antiphasis is 'the apophasis opposed to the kataphasis and the kataphasis opposed to the apophasis' (cf PG 94:653).

Formal definitions do not strictly follow the Aristotelian (and thus Damascenian) conception of antiphasis as the opposition of kataphasis and apophasis; but they are nevertheless Aristotelian. The terminology of kataphasis and apophasis, for example, is not maintained, and we see an emphasis on logical impossibility as the criterion for truth and falsity. For example, in his Symbolic Logic (fifth edition), Irving M. Copi gives the following sentential definition:

> One statement is said to contradict, or be a contradiction of, another statement when it is logically impossible for them both to be true… A statement form that has only false substitution instances is said to be contradictory or a contradiction, and the same terms are applied to its substitution instances. The statement form $p\ {\sim}p$ is proved a contradiction by the fact that in its truth table only **F**’s occur in the column that it heads (1979 [1954], p. 28; author’s emphases).

Such a statement form may have any number of different substitution instances, each of which is equally logically impossible and (thus) contradictory. But what will be definitive has already been noted by Aristotle. Opposing predicates cannot be ascribed to the same subject at the same time and in the same respect (cf. Cat. 10.13b, 33–5). Thus, it is also the case that a statement of the form $((x)(\Phi x \supset \Psi x)\ (\exists x)(\Phi x\ {\sim}\Psi x))$ or $((x)(\Phi x \supset {\sim}\Psi x)\ (\exists x)(\Phi x\ \Psi x))$, for example, is a contradiction. This kind of contradiction is often portrayed in logic texts[9] as a diagram composed of four statements (two contraries, two subcontraries and two contradictions), which is known as the square of opposition. It is the proverbial AO (universal affirmation [all S are P] plus particular negation [Some S are not P]) and EI (universal negation [No S are P] plus particular affirmation [Some S are P]) diagonal pairs that are contradictory. Such contradictions are defined in terms of logical entailment, when both p and q entail the other’s negation. A statement p logically entails the negation of q (i.e. ~q), and q logically entails the negation of p (~p). That is, $((p \supset {\sim}q)\ \ (q \supset {\sim}p))$. Thus, both ‘all men are mortal’ (p) entails ‘some men are not mortal’ (q) is false (~q), and vice versa.[10]

So what is contradiction a speaking against? Several answers are possible. It is speaking against in the sense of the opposition of kataphasis and apophasis, and vice versa, or a speaking against a subject predicate relation, or a propositional truth, or a speaking against speaking, etc. Or, as our discussion has been anticipating, it is a speaking against the law of non-contradiction (LNC). A contradiction is a speech act that instantiates LNC violation. We must turn back to Aristotle for an explanation of what we mean by LNC. He presents three versions of it in Met. These appear at 4.3.1005b, 19–20, 4.3.1005b, 24 (cf. 29–30), and 4.6.1011b, 13–20. In the latter of these sections, he speaks of the LNC as ‘the most indisputable of all beliefs’. And the formulation runs as follows: ‘contradictory statements are not at the same time true.’ But the LNC in Aristotle is primarily a principle of being. In Met. 3, for example, he holds that ‘a thing cannot at the same time be and not be’ (2.2, 29–30; cf. 4.1005b, 23–26 and 11.1061b.5ff). Thus, we get contemporary formulations such as ‘nothing in reality can correspond to a logical contradiction.’ This is more basic to Aristotle’s notion of the LNC. For it is because a thing cannot both be and not be at the same time that kataphatic and apophatic propositions, which have the same subject and predicate, cannot both be true. Thus, in propositional calculus, ${\sim}(\ p\ {\sim}p)$.

Not every Ancient Greek philosopher bought the notion of LNC. Heraclitus, for example, seems to have promoted just the opposite. His was a position with which Aristotle was not sympathetic. He expresses disagreement with Heraclitus’ tolerance of contradiction in Topics (8.5, 159b, 31–3), Physics (1.2.185b, 19–25), Metaphysics (4.3.1005b, 23–4; 4.7.1012a, 24–5; 11.5.1062a, 32–4). Beginning with the latter text, he says (see Met. 4.3.1005b, 23–4) that ‘it is impossible for anyone to believe the same thing to be and not to be, as some think Heraclitus says.’ And in 4.7.1012a, 24–5 he speaks of ‘the doctrine of Heraclitus,’ which is, in his view, ‘that all things are and are not.’ And that, Aristotle says, ‘eoike…hapanta alēthē poiein,’ ‘seems…to make everything true.’ Turning now to the Phys. 1.2.185b, 19–25, he argues that ‘if all things are one in the sense of having the same definition, like ‘raiment’ and ‘dress,’ then it turns out that they are maintaining the Heraclitean doctrine, for it will be the same thing ‘to be good’ and ‘to be bad, and ‘to be good’ and ‘to be not good,’ and so the same thing will be ‘good’ and ‘not good,’ and man and horse; in fact, their view will be not that all things are one, but that they are nothing; and that ‘to be of such-and-such a quality’ is the same as ‘to be of such-and-such a size.’ Heraclitus’ position is similarly referenced in Top. 8.5, 159b, 31–3. Aristotle conjectures in Met. 11.5.1062a, 32–4 that

Heraclitus might have been argued out of his delusion if someone had questioned him and 'forced him to confess that opposite statements can never be true of the same subjects.' Jonathan Barnes [5] has maintained that

> Heraclitus' central contention, the Unity thesis, is inconsistent; it flagrantly violates the Law of Contradiction; hence it is false, necessarily false, and false in a trivial and tedious fashion (p. 60).[11]

This seems to be consistent with, though somewhat more strongly worded than, Aristotle's own view of Heraclitus. G. S. Kirk has suggested that Aristotle 'seems entirely to misrepresent the opposite doctrine, or at any rate to subject it to a kind of criticism which is really irrelevant to it.' For, in Kirk's view, Heraclitus' concept of 'the same' is not synonymous with 'identical' ([20], p. 19). This may or may not be the case, although it is likely that Heraclitus had a more nuanced view of the matter. But it is not a significant issue as regards the LNC, as Laurence R. Horn has suggested it is in his 'Contradiction' in SEP [30], because whether Aristotle correctly regards Heraclitus on this in particular, he is nevertheless justified in viewing him as one who does not share an understanding of contradiction that is consistent with his own. For whether Heraclitus maintains that p = ~p, or had in mind some fine distinction that is not explicit in his fragments, he seems certainly to have maintained, as Hegel later thought, that p ~p. And, for Aristotle, that position is the real problem with Heraclitus. For if the most fundamental alternatives are motion and immobility, rather than one and many, as regards kataphasis and apophasis, and if it is the case that, as Plato maintains in Theatetus, 'if all things are in motion, every answer to any question whatsoever is equally correct' (183a[12])—with which Aristotle seems to agree when he says 'if all things are in motion, nothing will be true; everything will be false. But it has been shown (Met. 4.7.1012a, 24–5?) that this is impossible'—, then contradiction is, for Aristotle, an instance of such lack of permanence, and, as he sees it, the problem with allowing inconsistency of the form p ~p, then, is that, as he puts it in Met. 4.7.1012a, 24–5, it 'seems… to make everything true.' And that is perhaps the earliest expression of something like what comes to be known as the ECQ. It would be saying too much to affirm that this is in fact a position that Aristotle held; for it is presented in Met. 4 as something that eoike, or seems, to be the case. He may have suspected that tolerance of (some) contradiction would be explosive; but he did not clearly endorse that view.

The LNC is important for Aristotle's logic, but it does not necessitate adherence to ECQ. However, commitment to both runs deep in analytic philosophy. William Stanley Jevons' comment in his Elementary Lessons in Logic (London: Macmillan and Co., 1957 [1870]) expresses a somewhat weaker perspective than what becomes generally accepted in analytic philosophy. What he says is, in fact, quite in accord with Aristotle. As he sees it,

> It is the very nature of existence that a thing cannot be otherwise than it is; and it may be safely said that all fallacy and error arise from unwittingly reasoning in a way inconsistent with this law. All statements or inferences taken which imply a combination of contradictory qualities must be taken as impossible and false, and the breaking of this law is the mark of their being false (118; italics mine).

But it is this sort of perspective that is nevertheless the source of the modern ECQ notion. By the time of Russell and Frege, ECQ becomes a philosophical dogma for analytic thought, and, thus, the source for the kind of inconsistency intolerance of the form indicated by Barnes above. Many such examples could be cited. One figure who has undoubtedly played a leading role in promoting it is W. V. Quine. Speaking to the suggestion that we 'reject the law of non-contradiction and so accept an occasional sentence and its negation both as true,' Quine says '[m]y view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation, '~', 'not;' but surely the notation ceased to be recognizable as a negation when they took to regarding some conjunctions of the form p ~p as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject' (in 'Deviant Logics' chapter 6 of his Philosophy of Logic [Prentice Hall: 1970], p. 81). For Quine, and many others, intolerance of inconsistency, especially contradiction, is a cardinal philosophical dogma. That, as we noted above, is derived in large part from Aristotle's own view of the LNC.

The two Orthodox beliefs mentioned above are instances of contradiction; thus, they violate the LNC. But there are two types of contradiction amongst writers in the Orthodox tradition. Most writers have held these beliefs in such a way that inconsistent predicates are affirmed of the same subject. Others, most importantly Pseudo-Dionysius, both affirm and deny inconsistent predicates with regard to the same subject. The first type of contradiction takes the form p ~p, where the contention is that this proposition is true. The other type is of the form ((p ~p) (~p ~~p)). Thus, Pseudo-Dionysius maintains that God is one and three and denies that God is one and three. This Pseudo-Dionysian form, though enormously (albeit at times confusingly) influential is not the conciliar, and, thus, technically Orthodox position. That is the simpler proposition that God is both one and three. In the terminology of the councils (i.e. Nicea 325 and Constantinople 381) Father, Son and Holy Spirit are homoousios, which is taken to mean that God is mia ousia kai treis hypostaseis, one essence and three persons (as later formalized at Constantinople in 553). This is the form the contradiction takes in the PGT. In 'Letter Six: Contradiction' (pp. 106–23), we find Florensky assuming the paraconsistent logic of Orthodox thought, but also trying to justify it. For if the LNC is true in the sense that isolated contradictions are never to be tolerated, then the Orthodox theory of God is false; for the logic supporting it would be faulty.

We have just said that it is a paraconsistent logic that produces the Orthodox theory of God. That may not be obvious. What would that kind of logic look like in this situation? The logic behind the Orthodox theory may be summarized as follows. First of all, it is a theory of simultaneous unity and distinction in God, namely the contention that God is both one and three. According to that theory, God is one essence in three persons. In earliest Christian thought, this took the form of triadic 'subordinationism.' The most notorious form of that view was Irenaeus of Lyons' 'two hands' theory. During the Arian crisis of the fourth century, Athanasius of Alexandria and the Cappadocian Fathers successfully argued for a triadic view that included the concept of co-equality. However, experience of divine behavior (especially an instance such as the baptism of Christ [theophany], e.g.), which had formed an integral part of its Trinitarian theory, indicated that there is either one God or three gods. Hence, Orthodox theory was not consistent with all of its experience. So in terms of the formulation of its theory of God not everything about experience was inferred. And viable options—monarchianism, arianism, pneumatomachianism (or 'macedonianism'), tritheism,[13] e.g.—any one of which would have been a sensible inference according to the LNC, were rejected. Therefore, the inferential methodology used was paraconsistent. Florensky formally justifies this sort of paraconsistency in rejecting the reductio. And in doing so, he formalizes the dialtheism of Orthodox thought. In writing the PGT, Florensky was influenced by Jevons (and many others). This is explicit in the text of 'On the Methodology of the Historical Critique' (PGT, pp. 384–89), where Jevons is twice (PGT, p. 384, 388) quoted and mentioned by name. Much earlier in the PGT, in 'Letter Six: Contradiction' (loc. cit.), the reader is referred to Jevons (among several other logicians—Poretsky, Peano, Schröder, Russell, Couturat, etc.) in two notes. But the LNC is not a logical dogma for him, as it is for Jevons and others. Rather tolerance of inconsistency means, very explicitly, tolerance of противоречие (protēvoryechēye) or антиномия (antēnomēya), 'contradiction' or 'antinomy.'[14] What he has in mind in particular is propositions of the form p ~p, which is what he speaks of as the antēnomēya P (cf. PGT, 112–3). He justifies embracing (at least some) contradictions on his rejection of the reductio. His analysis of the reductio shows that (what we now call) classical logic is explosive. For, according to his analysis, p ~p is derivable using the reductio form and a few basic replacement rules. He calls p · ~p 'the antinomy P.' Thus, P = (p ~p) = V, where 'P' is a proposition with two contradictory terms (or a class whose members mutually exclude one another), and V is the truth truth-operator. An antinomic proposition, he says, 'contains thesis and antithesis, so that it is inaccessible to any objection... [and] above the plane of rationality' (PGT, 113). Truth is antinomic for him. But that is not the same as saying that any contradiction is true. He is very explicit that the antinomy P is synonymous with contradiction. That term and the meaning we can now associate with it, gives us the fullest sense of what he means by ‘antinomy,’ namely speaking against. And that is essentially how we would understand it here were we to interpret it in a strictly etymological manner. For in that sense, antinomy means ‘against (the) law’ (from anti and nomos). Lexically, we find ‘conflict of laws’ and ‘contradiction between laws’ (LSJ). In a philosophical context, it is natural to understand ‘law’ in terms of the ‘laws of thought,’ specifically the LNC.

Some writers have been hesitant to affirm this. In his Pavel Florensky: A Metaphysics of Love, for example, Robert Slesinski says that ‘the requirements of conceptual clarity and terminological rigor demand that he be faulted for his poetic license and propensity for literary flourish,’ specifically as regards his synonymous use of the terms antinomy and contradiction. For they ‘denote different things,’ Slesinki asserts, ‘and should not be confused.’ In his view, “antinomy” would have been the better, more properly nuanced choice, even though it connotes the idea of contradiction’ (p. 147). In addition to speaking of it as a contradiction, the OED defines ‘antinomy’ as a ‘paradox.’ ‘Paradox’ comes from para and doxa. The preposition para here (as in the term paraconsistent) takes the meaning ‘contrary to’ or ‘against.’ The term doxa from dokeo, most likely means ‘expectation.’ It is commonly taken to mean ‘opinion,’ which is not wholly inaccurate. But a paradox is not what it is because of misalignment with opinion per se. It seems, rather, to be a matter of expectation. Something is a paradox (Zeno’s Achilles’ paradox, e.g. See Phys. 4.9.239b14–29) because it does not (seem) to be consistent with expectation. Indeed, one expects the quicker runner to overtake the slower one. But this meaning for ‘antinomy’ does not give us a more accurate definition. Since it is rather more epistemological, it may even complicate matters. And, in any case, an antinomy, as we use the term, is not merely something that goes against expectation. The OED gives a rather different meaning for ‘paradox’—‘a seemingly absurd or self-contradictory statement or proposition that may in fact be true.’ That is the way the term is commonly used, and it also has the merit of being about speaking rather than thinking. But the only significant nuances here are the qualifications ‘seemingly’ and ‘may in fact be true.’ And those are qualifications Florensky is not making. Nor are they consistent with Orthodox dogmas. The term dogma also derives from dokeo. The lexical meaning of import for dogma is that of ‘a resolution’ or ‘decree.’ And concerning truth in particular, the Trinitarian and Christological dogmas of Orthodox thought are, thus, reckoned to be true, in spite of the obvious inconsistencies. They are not maintained as paradoxes; nor (therefore) are they held to be antinomies in the sense indicated by the OED. For Florensky, the antinomy P is robust. It is a contradiction. Moreover, as he reads it, the Trinitarian and Christological dogmas are too. But why would he want to assert this? Does that claim not implicate falsity? This seems to be Slesinski’s concern. For, speaking of the dogmas as contradictions seems to hyperbolize the indigenous inconsistency. For insofar as ‘Christ is God,’ then it follows that ‘Christ is not a man.’ But insofar as ‘Christ is a man,’ then it follows that ‘Christ is not God.’ And, similarly, insofar as ‘God is one,’ then it follows that ‘God is not three.’ But insofar as ‘God is three,’ then it follows that ‘God is not one.’ And, furthermore, insofar as ‘All men are mortal’ and ‘Christ is a man,’ then ‘Christ is mortal.’ But insofar as ‘No God is mortal’ and ‘Christ is God’, then ‘Christ is not mortal.’ And from this it follows both that ‘All men are mortal’ and ‘At least one man is not mortal’ and ‘No God is mortal’ and ‘At least one God is mortal.’ However, this is not hyperbole; it is clarity. Moreover, the dogmas are of the form $P = (p \sim p) = V$. In other words, P = ‘Christ is God’ and ‘Christ is man;’ and P = ‘God is one’ and ‘God is three.’ And, P = ‘All men are mortal’ and ‘At least one man is not mortal;’ and P = ‘No God is mortal’ and ‘At least one God is mortal.’

In terms of the language of contradiction, however, Florensky tends to favor Kantian and Hegelian terminology rather than the Aristotelian and (later) Orthodox use of kataphasis and apophasis. The antinomy P, as he puts it, is composed of the ‘thesis p’ and the ‘antithesis ~p.’ P is true if it cannot be shown that the thesis and antithesis are false. In other words, if it can be shown that thesis p and antithesis ~p are false contraries, then P is not true. If it is a bona fide contradiction, then it is true. He borrows the term ‘antinomy’ from Kant’s use of it in book two, chapter two of the second division of the first Critique (esp. A427/B455 ff.), crediting him for its 'very late origin.'[15] For the origin of the concept, he turns to Heraclitus. And his reading of Heraclitus is consistent with Aristotle's. He finds him to be a proponent of the opposition of kataphasis and apophasis; but, for him, this means that Heraclitus is a champion of the concept of truth as contradiction, and (as Russell does in his 'Mysticism and Logic') presents several of the fragments of Heraclitus to show this. He also mentions several other figures who, in his view, were proponents of antinomy. This begins what amounts to a general (and sometimes effusive) summary of the history of philosophy and Orthodox thought as it pertains to antinomy. After Heraclitus, he makes brief reference to the Eleatics (Xenophanes, Parmenides, Zeno, Melissus[16]), Plato and Nicholas of Cusa's coincidentia oppositorum; and then merely names Hegel, Fichte, Schelling, Renouvier and the 'pragmatists' (with reference to a previous note [77] on Pascal's wager in which he cites numerous texts of and on pragmatic philosophy). Next he turns to Job as a scriptural example of antinomy. In that paragraph, he also relates the concept of antinomy with the concept of mystery (taina) and the act of silence (molchaniya). He adds that

> The mysteries of religion are not secrets that one must not reveal. They are… inexpressible, unutterable, indescribable experiences, which cannot be put into words except in the form of contradictions… (PGT, p.117).

And so the next important proponent of antinomy he mentions is Orthodox dogma, which he contrasts with heresy. In his view, whereas dogma is antinomic, heresy chooses sides, either the thesis p or the antithesis ~p. In this way, heresy is rational, but false. Orthodox rationality, however, acquires the truth by means of a kind of rational humility. Earlier in the chapter (cf. PGT, p. 109), he says that o podvige rassudka est' vera, 'the podvig of reason is belief/faith'. The term podvig indicates a bold feat or great deed; it is commonly used in Orthodox thought to designate ascetic and spiritual practices. In his The Path to Salvation: A Manual of Spiritual Transformation,[17] the prominent Russian spiritual writer, Theophan the Recluse, speaks of podvig as consisting of 'self-opposition and self-forcing' (p. 208). Here Florensky uses it because, in his view, antinomy is vne-rassudochnogo, 'extra-rational', (rather than irrational or non-rational). The podvig very, 'podvig of belief/faith', as he also calls it (cf. ibid.), is the method of attaining truth beyond the LNC. In the podvig of which Theophan speaks, one forces oneself to engage in exercises such as fasting and alms giving or confession and communion. Such practices are thought of as spiritual exercises. From Florensky's perspective, belief/faith is the fundamental podvig, the most elemental spiritual exercise. It is a difficult feat, and comes to something along the lines of what Paul has in mind when he speaks of tēn logikēn latreia, 'rational worship,' and of being metamorphousth tē anakainōsei tou noos (ēmōn), 'transformed by the renewing of your mind' (Rm. 12.1, 2).

From Florensky's perspective, and here he is very much in tune with the logic of Orthodox thought, contradictions are eliminated not by disjunctive reasoning but by conjunctive reasoning. Rather than either kataphasis or apophasis being true and the other one being false, Florensky promotes the idea that, concerning the tajny religii, 'mysteries of religion,' the podvig of both-and logic, of believing in spite of opposition, achieves coherence supra-rationally. This is apparently what he finds in the authors he mentions. And it is, as indicated in the brief comments about Plato, a kind of cel'nogo rassudka, which Jakim renders as 'integral rationality' (PGT, p. 116).[18] The term airesis, from which we get the English 'heresy' (and the Russian eres'), has the lexical meanings of 'conquering,' 'taking for oneself' and 'choosing;' (it can, of course, also designate a 'sect'). These meanings have one main thing in common. For conquering, taking and choosing each has to do with something, something that is part of other things. If there is something called 'integral rationality,' then, in Florensky's mind, there is also a heretical rationality, a form of reasoning that opts for one part or another, either p or ~p. But it is necessary to keep in mind, in spite of fulsome comments such as 'contradictions are in everything' (PGT, 116), that integral reasoning, since it is tolerant of inconsistency, is not explosive. It may be the case that contradiction is the hallmark of truth, but that does not mean that just any contradiction turns out to be true. Spinoza's pantheism is an example.

Thus, there are no grounds for showing that Florensky is as unavoidably committed to the truth of the contradiction 'Pavel is his own daddy,' for example, as to something like 'God is both one and three.' Integral reasoning does not imply the truth of just any contradiction. For some contradictions are intuitively intolerable. They are, we might say, prima facie false. Claiming that a man is his own daddy immediately smacks of incomprehensibility and error. And other contradictions are equally erroneous, though perhaps not as obvious. Consider, for example, 'The universe is both geocentric and heliocentric' and 'One of the five solids is a one-dimensional figure known as the round-square or dodecahedron.' Whereas 'Pavel is his own daddy' is intuitively intolerable, these latter two seem to require some pre-existent knowledge. But they are just as false. Provided we have a fair understanding of astronomy and geometry, we can quickly determine that. These two types of contradictions are absurd and falsifiable. A third category consists of counter-intuitive claims—such as the Liar (or Epimenides) Paradox, the Chalcedonian definition or the Hilbert Hotel Paradox (where it is true that the hotel is both full and has vacancies)—that seem to be true. If such claims are accepted as being inconsistent but true, then they are contradictory in a manner quite distinct from the above two types of contradictions. That difference consists in the uncovering of mystery. Intolerable contradictions (absurd and falsifiable claims) are not able to do that. It is this sort of tolerable contradiction—integral instances of LNC violation—that interests Florensky. By way of concluding the chapter we have been discussing, he lists eleven of them. Each concerns Orthodox belief and practice. The top two are the ones we have already mentioned, namely the Orthodox doctrines of God and Jesus Christ. The others we do not need to rehearse here. We must only underscore that the list is composed of contradictions that are relevant to Orthodox belief and practice. The first two are the core of Orthodox thought. There is a unique feature to these contradictions, which is especially explicit in these first two examples. That feature concerns an analytical criterion of integral contradiction, which we may call the principle of simultaneous union and distinction. For that gives a more refined sense of what sort of mystery is the subject of accepted instances of inconsistency in Orthodox thought.

A true contradiction is a mystery. This is the difference between false and true contradictions. The distinct characteristic of mystery is the difficulty one experiences in trying to understand and explain it either as true or false. A false contradiction, however, can be either known or shown to be so quite easily and conclusively. No mitigating factors remain. The claim 'Pavel is his own daddy' is not difficult to understand or explain; we understand (and could explain) that it is erroneous. The truth-value of something like 'God is both one and three' is similarly determinable in that it is not demonstrably false. Why? To answer this, we must introduce a distinction. For in Orthodox thought metaphysics is not just a matter of being per se, as is the proposition about Pavel; it is rather about two classes of being. On the one hand, there is created being; and, on the other, there is uncreated being. In general, in Orthodox thought the LNC applies to created being, but not to uncreated being. However, there are indications in the thought of the Eastern Fathers that it is not applicable in created being. One obvious example, similar in form to 'married bachelor,' is the Orthodox belief in Mary as Virgin Mother; the most evident and universally acknowledged instance where it is not wholly applicable, though, is in the incarnation. Back to the main point, though, the claim 'God is both one and three' is not absurd or demonstrably false because the subject, according to Orthodox thought, is uncreated.

So, what is the inference mechanism that allows affirmation of inconsistency in isolation? Florensky gestures toward mystery. Not just any mystery though. The mystery Florensky sees as distinguishing a true contradiction from its false cousins induces silence; it gets one to the point of being speechless. Not just speechless. Lack of speech is understood in terms of prayer and worship, and in terms of ineffability. Moreover, that sort of mystery must be akin to hope. But again, Florensky has a particular kind of hope in mind. The only hope that matters is his concern. If inconsistency is to be tolerated in isolation, then it must provide hope in the face of death. The source of that hope must be love that is victorious over the enemy of being: death. Thus, measuring mystery in Orthodox thought requires not just critical evaluation of the logic of propositions; but also, and most essentially, it requires love.

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Notes

1. *Such as the Hindu Upanishadic and Vedantic philosophies and the Mādhyamika philosophy in Buddhism.*
2. See Vasiliev 'Na chastichno suzhdenijah, na treugol'nike protivopolozhnostej, na zakone iskljuchennoj chetverti' (1910) and 'Imaginary (non-Aristotelian) Logic' in [28] (= English summary of 'Mnimaja (non-Aristotelevskaja) Logika'[1912]); [1], [2], [3].
3. On Vvedensky and Lapshin see [21], pp. 163–70; [31], pp. 678–87 and 687–95. Both are also mentioned in [10]. See also [26] for translated sections of Vvedensky's *Logika kak chast' teorii Poznaniia* (*Logic as Part of the Theory of Knowledge*) and Lapshin's *Zakony Myshleniia i Formy Poznaniia* (*The Laws of Thought and Forms of Cognition*).
4. Following [25]. For the Greek of *Metaphysics* I use his [24].
5. Translated by Thomas A. Carlson and published in 2001 by Fordham University Press.
6. This is E. M. Edghill's translation in [25], p. 43. Line numbers here (and for *Cat.*) follow the Greek text in LCL 325 (=[8]).
7. John's dates are difficult to determine exactly, and they often differ considerably. The dates I give here follow Robert Volk's 'John of Damascus' in [9] (1998=*Lexikon der antiken christlichen Literatur*), pp. 338–9.
8. Also known as *Dialectica*.
9. For example: [17], p. 7; and [6], pp. 267–272; Copi (1979 [1954]), pp. 66–8. The original source is *An. Pr.* 1.2, 1–25 and *de Int.* 17b, 17–26.
10. See [29]; and Peter Geach 'Contradictories and Contraries' in chapter two ('Traditional Logic') of [16], p. 70ff.
11. See also his [4].
12. LCL 164 (= [15]).
13. There were proponents of each of these inferences. Respectively: Noetus, Praxeus, Sabellius, Photinus, Marcellus of Ancyra (=modalists), Theodotus, Paul of Samosata (=adoptionists, psilanthropists, dynamic monarchists); Arius, Eustathius of Sabastia (whom Basil speaks of as the leader of the pneumatomacians [*ep.* 263.3]); and tritheism appears a bit later in John (Grammaticus) of Philoponus (and, much later in the west, probably Roscellinus and Gilbert de la Porrēe, who, incidentally, was influenced by Boethius).
14. According to the new (post-revolution) orthography.

15. On what he calls *perezhivanii antinomichnosti*, he refers the reader to his [11]. See also his [12].
16. And perhaps others (cf. *Sophist* 242c–d).
17. Translated by Fr. Seraphim Rose and the St. Herman of Alaska Brotherhood (Platina: St. Herman of Alaska Brotherhood, 1998).
18. Precursors to this concept include [18], v. 1, p. 327 (*cel'nosti razuma*); [19]; and [27].

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Logic in Opposition

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Abstract:
It is claimed hereby that, against a current view of logic as a theory of consequence, opposition is a basic logical concept that can be used to define consequence itself. This requires some substantial changes in the underlying framework, including: a non-Fregean semantics of questions and answers, instead of the usual truth-conditional semantics; an extension of opposition as a relation between any structured objects; a definition of oppositions in terms of basic negation. Objections to this claim will be reviewed.

Keywords. consequence, existential import, multifunction, negation, opposite, opposition

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1. Introduction

The paper wants to do justice to the central contribution of opposition to the way meaning is currently formed and conveyed. For this purpose, let us tell some words about the meaning of "meaning" while turning to the very content of opposition, from Aristotle's works to a general theory between logic, ontology, and algebra.

1.1. Opposition and Meaning

Meaning has to do with information, and information is not a ready-made collection of related objects. Moreover, existence does not seem to be so a crucial property of an object once information has more to do with how people interact with each other. Theses precisions may help to bring some light upon the philosophical background of our logic of opposition, where the central concept of "truth" has to be treated very cautiously in an intersubjective sense of accepted information.

That a formal semantics equally applies to different categories of things like individuals, concept or sentences entails that our so-called "logic" of opposition lies between formal ontology and algebra. However, it can be called a theory of meaning safely insofar as it relies upon any questions and answers liable to present something as a relevant information beyond the sole case of sentences. To put it in other words, the following semantics departs from the realist-minded notion of truth by relating meaning to the way in which any piece of information is given about a putative object. The more objects there are in a given local ontology, the more questions are to be asked in order to make order between them. Borrowing from the Goodmanian parlance [6], there are several ways of making worlds and, correspondingly, one and the same object can have a different meaning-in-a-model (a local ontology) according to the number of properties it is provided with.

That it *exists* is a thing; but another thing is that, according to us, existence is neither a necessary nor a sufficient condition to say anything meaningful about it. Therefore, one and the same object can belong to different worlds (or models) once different properties are assigned to it or, better, different perspectives are entertained to make a description of it. To push the line further, let us say that the so-called "actual" world is a maximal lexical field, i.e. a proper set of overlapping sets of information; each element of this common world can (and, indeed, does often) belong to different such subsets that are to be compared with "possible worlds", i.e. different perspectives (lexical fields) from which they are entertained as valuable pieces of information.

1.2. Plan of Work

The theory of opposition is investigated and revisited in the present talk. This will be made in two main steps, defensive and constructive in turn. (1) Against a widespread opinion, it is argued that such a theory is not just an old-fashioned legacy of Aristotle's traditional logic that would have definitely failed because of the problem of *existential import*. (2) Beyond the current view that logic is a theory of consequence, it is suggested that opposition is a more basic relation encompassing Tarski's consequence as a particular case of opposition. (3) Objections to this counterintuitive view of opposition will be reviewed and lead to a more dialogical view of the logical discipline: the aim of logic is not so much preserving truth than expressing structured differences.

Logic as a theory of difference will be defended as follows.

(1) According to Aristotle's logical legacy [1], there are four kinds of logical opposition between universal and particular propositions: *contrariety*, *contradiction*, *subcontrariety*, and *subalternation*.

After defining these, attention will be paid first upon the so-called problem of existential import; the logical square of opposition is said to be invalidated once the predications are about empty terms, leading to a radical depreciation of the theory of opposition because of its allegedly limited application and dependence upon some preconceptions of traditional logic. Against this definite view, it is shown that existential import does not invalidate the logical square under some alternative formalization of the propositions [4].

(2) Then the concept of opposition is abstracted from its historical context and developed into a set-theoretical approach [14,15,16]. Firstly, opposition is given as a binary *relation* between *structured objects*. Secondly, a correlated theory of opposites depicts oppositions as a relation between a relatum and its opposite. Thirdly, a non-Fregean semantics leads to a Boolean calculus of opposites: *Question-Answer Semantics* (hereafter: **QAS**), where the logical value of any structured proposition is an ordered set of answers to corresponding questions. In the case of logical oppositions, the meaning of structured propositions is afforded by questions about their *disjunctive normal form*. A Boolean algebra of the classical oppositions follows from it and matches with Piaget's INRC Group [12]. Fourthly, the crucial role of *negation* accounts for the oppositional roots of logical consequence, and its oppositional nature is justified by claiming that subalternation proceeds as a double mixed negation.

(3) Finally, a number of objections will be addressed about this revisited theory of opposition. These can be summed up by the following questions:
(a) Can opposition be something else than a relation of incompatibility? (c) Isn't subalternation a restrictedly standard view of logical consequence? (d) Can one set up a proper calculus with the opposite-forming operators?

A way to reply to this set of objections requires an alternative view of logic: not a theory of truth-preserving consequence, but a theory of difference-forming negation. A way to uphold this trend within **QAS** requires the epistemological primacy of *negation* upon truth. The variety of logical negations must be distinguished from the unique operator of *denial* for every opposite relation between structured objects.

2. The Historical Background of Opposition

Two reasons may be advocated at the least to show that the theory of opposition is on a par with Aristotle's logical works. For one thing, the famous "square of opposition" is currently assigned to the philosopher's name, although it has been argued elsewhere (e.g. in [14]) why Aristotle never mentioned any such figure in his logical writings. On the other hand, each of the well-known relations of opposition finds its roots in Aristotle's texts, too. This is not the whole story, however, in the sense that a properly logical theory of opposition can be displayed without resorting to traditional logic. In this respect, a formal device can be used to set up a Boolean algebra of opposition which doesn't take into account any other information than logical values.

Let us return to the historical background of logical oppositions, however, in order to see more clearly how an algebraic logic of oppositions can be freely abstracted from the Aristotelian theory of quantified propositions while embracing it altogether.

2.1. Definitions

Aristotelian oppositions are characterized by some constraints upon the truth-values of related propositions *a* and *b*.

Proposition 1
a and *b* are *contrary* to each other iff they cannot be *true* together.

Proposition 2
a and *b* are *contradictory* to each other iff they cannot be *true* together and cannot be *false* together.

Proposition 3
a and *b* are *subcontrary* to each other iff they cannot be *false* together.

Proposition 4
b is *subaltern* to *a* iff *b* cannot be false whenever *a* is true.

A number of questions arise from this preliminary presentation, including the three following ones. First: why does one deal with logical oppositions in the form of a square, i.e. why should one stick to four logical relations among its six edges (four straight lines and two diagonals)? Next: does it make sense to talk about subcontrariety and subalternation within a theory of *opposition*? Aristotle depicted the former in terms of "verbal oppositions" (see e.g. [3], p. 416) while ignoring the latter as an opposition altogether, after all. Last, but not least: what of non-classical negations with respect to the theory of oppositions? While Aristotle clearly linked opposition and negation through the so-called laws of *non-contradiction* (a proposition and its negation cannot be true together) and *excluded middle* (a proposition and its negation cannot be false together), contradiction is the only kind of opposition that relates to negation from this classical (bivalent) perspective.

Another focus is in order before answering these questions in the sequel, namely, one of the logical problems that led to the historical fall of the theory of opposition.

2.2. Existential Import

According to the so-called problem of *existential import*, the logical square of opposition is made invalid by a standard, truth-functional semantics once propositions refer to empty names, i.e. dummy individuals that don't exist (like "the present king of France", "griffins", and the like). If so, then its applicability is restricted to non-empty models and thereby weakens the scientific relevance of a logical theory of opposition. Such a semantic difficulty has to do with the way truth-values are assigned to propositions, since a predication like "S is P" assumes for its truth that S be instantiated by at least one individual (x, say).

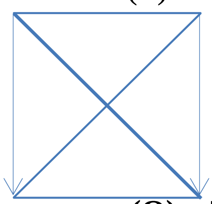
A modern formal translation of Aristotle's traditional logic turns predications into quantified propositions like "$(\ldots x)(S(x)\ \ldots\ P(x))$", where the blanks are to be filled by quantifiers (either universal or existential) and logical connectives (either conditional or conjunction). Except for the ambiguous case of singular propositions, the result is a clear correspondence between traditional and modern formulas.

Proposition 5

Formulas from traditional logic can be translated in modern first-order logic as follows. (A) Universal affirmative: "Every S is P" := $(\forall x)(Sx \supset Px)$; (E) Universal negative: "Every S is not P" := $(\forall x)(Sx \supset {\sim}Px)$; (I) Particular affirmative: "Some S is P" := $(\exists x)(Sx \wedge Px)$; (O) Particular negative: "Some S is not P" := $(\exists x)(Sx \wedge {\sim}Px)$.

Let us consider the sentence "Some griffins are nice". The truth-value of this particular affirmative relies upon whether there is some griffin which happens to be nice. But there cannot be some such creature, for no griffin exists at all. Hence the first conjunct Sx is made false, and so is the entire conjunctive proposition. Let us write by v(I) = F the case that the I-proposition is false. This entails that its contradictory, i.e. the corresponding universal affirmative, is true, according to the definition of contradictories just given above: v(E) = T. This sounds intuitively right, since no griffin could be nice once no such creature exists. But the tricky point is about its subaltern, i.e. the related particular negative to the effect that some griffin is not nice. Such a proposition cannot be true whenever no griffin exists, so that v(O) = F. The whole set of logical oppositions is thus at odds with their aforementioned definitions, as witnessed by the following invalid square and its troublesome relation (in bold face).

***v*(A) = F** v(E) = F



v(I) = T ***v*(O) = T**

A number of replies have been proposed to settle this problem, namely: restricting the existential import of propositions; discarding their formal modern interpretation; invalidating the square as it stands, otherwise. Our own solution would consist in changing the formalization of particulars, as argued in a recent paper (see [4]); in a nutshell, our point is that the contradictories of universals should not be rendered in the form of existential propositions whose truth-conditions require the existence of their subject-term. Whatever the explanatory value of this formal reply may be, it helps to save the square and enhances its scientific value within the realm of logic.

Once the square of quantified propositions is restored, we can push the line further by abstracting from the category of sentences Aristotle was strictly concerned with. Logical values are the essential information required to define logical oppositions, indeed, and any sort of meaningful object is included in our discussion. But to do so actually requires another formal semantics than the truth-conditional one.

3. A formal theory of opposition

The subsequent formal semantics makes a primary distinction between oppositions and opposites, before defining their features by means of Boolean bitstrings. Just as Tarski suggested an abstract view of consequence as either a relation between sets of formulas or an operator [20,21], the same treatment will be reserved to the logical concept of opposition.

3.1. Opposition as a relation

It is taken to be granted that opposition proceeds as a relation between objects, irrespective of how many and what these are exactly. Although the mainstream theory of opposition usually refers to the binary relation between propositions (as e.g. in [14]), it will be argued in the following that our proposed semantics needn't apply to propositions and equally applies to individuals, concepts, or whatever does make sense by means of a question-answer game.

Proposition 6

An opposition Op is an ordered binary relation between any meaningful objects a and b: Op(a,b), such that it holds iff the 2-tuple of objects a,b satisfies Op.

It is worthwhile to note that Op has been restricted hereby to the arity $n = 2$, although more than two contrary oppositions can be related to each other satisfactorily. It is thus the case that e.g. necessary, impossible and contingent propositions are contrary to each other. Yet this is not the case for most of the other relations like, e.g., contradictories: if a is contradictory with b and b is contradictory with c, then a is not contradictory but, rather, identical to c. For take "white" as an instance of a; then its contradictory b is "not-white", and the contradictory c of the latter is "not-not-white", i.e. "white", while the contrary of "white" is "black". We will return to these peculiarities later on (see section 3.3).

For the time being, let us note not only that any n-tuple of a valid relation of opposition can be reduced to a set of 2-tuples (see [13] for a similar rationale with classical 3-ary connectives); but also, that these relations can be constructed through intermediary operations within a more fine-grained formal semantics.

3.2. Opposite as an operation

Taking the preceding example again, the concepts "white" and "black" stand into a contrary relation. We propose in the following to investigate the logical properties of "blacken", that is, the operation by means of which anything white is turned into something black.

Proposition 7

An opposite O is a mapping O(a) upon a relatum a of an opposition, such that it turns it into the second relatum b of the given opposition: Op(a,O(a)) = Op(a,b).

A logic of colours has already been set up by Jaspers (see [8]) in the same vein, where chromatic oppositions are displayed by a set of Boolean bitstrings that is going to be explained in the following semantics.

3.3. An algebraic semantics for oppositions and opposites

Our formal semantics has a twofold purpose: to afford a formal theory of meaning for any sort of objects from mere individuals to usual sentences; to set up this semantics with the help of Boolean algebra. While it is *a locus classicus* to say that only sentences make properly sense by their truth-conditions, the following leads to a more comprehensive "non-Fregean" semantics that characterizes the sense by means of questions and answers.

3.3.1. Question-Answer Semantics

A special attention is paid to the way in which information is conveyed about an individual, concept, or sentence; indeed, how they are depicted by some of their properties may have a deep influence on their general meaning. This leads to a question-dependent view of meaning, where the value of any given information relies upon the sorts of properties put into focus.

Our Question-Answer Semantics (hereafter: **QAS**) resorts to a non-Fregean theory of sense and reference, assuming that no reference is a truth-value. By doing so, **QAS** is on a par with Roman Suszko's critics of the so-called "Fregean Axiom" in [19].

Proposition 8

The *meaning* of any object a is determined by its sense and its reference, *sense* being a finite ordered set $\mathbf{Q}(a) = \langle \mathbf{q}_1(a),...,\mathbf{q}_n(a)\rangle$ of *n questions* about a (where $n \geq 1$) and *reference* being the set $\mathbf{A}(a) = \langle \mathbf{a}_1(a),...,\mathbf{a}_n(a)\rangle$ of corresponding *answers*.

The standard, truth-conditional semantics can be embedded as a special case of our question-answer framework, by using the words "true" and "false" as the metalinguistic predicates of specific questions among other ones. By contrast, our non-standard semantics results in a calculus of logical values while going beyond the prominent case of "truth-values".

3.3.2. Boolean algebra of oppositions

Given that m sorts of answers can be given to n questions, there are m^n possible values for each a. For instance, asking $n = 3$ questions about a and having $m = 2$ available answers yields a set of $m^n = 2^3 = 8$ logical values including $\mathbf{A}(a) = 111, 110, \ldots$ until 000. The number of such logical values is relative to the formal ontology within which a is presented; that is, it depends upon how many data are needed in order to be able to individuate a, i.e. to make it logically different from any other object in a given set. In a nutshell, n is a sufficient amount of questions iff $\mathbf{A}(a) \neq \mathbf{A}(b)$, assuming that these questions can characterize anything meaningful by a finite set of properties (i.e. the semantic predicates of a question). The perplexing cases of vague predicates and ensuing paradoxes should lead to a Boolean counterpart of infinite-valued matrices; but they won't be considered in the present paper.

It is worthwhile to note that the objects are not provided with a *single* value like "true" or "false" in **QAS**; rather, their reference amounts to an ordered combination of single *sub-values* that stand for each of the answers. We stick to the Boolean values 1 and 0 in the sequel, where $m = 1$ is a yes-answer and $m = 0$ a no-answer, while pointing out that a question-answer game needn't be confined into such binary answers. Let us call by a *bitstring* any such structured string of ordered answers; in the case of a Boolean algebra, each sub-value of a string takes either 1 or 0 and is thereby reminiscent of logical bivalence. At the same time, the m^n possible values of an object go largely beyond two cases whenever $m > 1$ and result in something analogous with a many-valued calculus of Boolean bistrings.

A calculus of logical oppositions is made possible by making use of bitwise operations.

Proposition 9
$\cap$ and $\cup$ are the operations of *meet* and *join* upon values 1 and 0, such that $1 > 0$. Then:
$x \cap y = max(x,y)$;
$x \cup y = min(x,y)$.

The Aristotelian relations can be rendered algebraically by asking questions about the compossibility of truth-values between any two propositions a and b. Assuming that every classical (bivalent) proposition a can be translated by a disjunctive normal form $\mathbf{A}(a) = \langle \mathbf{a}_1(a),\mathbf{a}_2(a),\mathbf{a}_3(a),\mathbf{a}_4(a)\rangle$, to characterize such a propositional opposition between a and b amounts to a questioning about their various compossibilities among $n = 4$ possible cases, namely: whether a and b can be true together; whether a can be true while b is false; whether a can be false while b is true; whether a and b can be false together.
More generally, oppositions go beyond the sole logical category of propositions and are to be defined in common terms of compossible yes- or no-answers for their arbitrary objects a,b, irrespective of the sorts of questions to be asked about them.

Proposition 10
Opposition $\mathrm{Op}(a,b)$ is a set Op = {CT,CD,SCT,SB} of relations to be defined:

10.1 by the logical values $\mathbf{A}(a)$ and $\mathbf{A}(b)$ of any two objects a and b such that, for any i^{th} question of the same question-answer game, these stand into a relation of:

contrariety $\mathrm{CT}(a,b)$	iff	$\forall \mathbf{a}_i$: $\mathbf{a}_i(a) = 1 \Rightarrow \mathbf{a}_i(b) = 0$
contradiction $\mathrm{CD}(a,b)$	iff	$\forall \mathbf{a}_i$: $\mathbf{a}_i(a) = 1 \Leftrightarrow \mathbf{a}_i(b) = 0$
subcontrariety $\mathrm{SCT}(a,b)$	iff	$\forall \mathbf{a}_i$: $\mathbf{a}_i(a) = 0 \Rightarrow \mathbf{a}_i(b) = 1$
subalternation $\mathrm{SB}(a,b)$	iff	$\forall \mathbf{a}_i$: $\mathbf{a}_i(a) = 1 \Rightarrow \mathbf{a}_i(b) = 1$

10.2. by the Booleans operations of meet and join, together with the logical values of *tautology* $\top$ (only yes-anwers) and *antilogy* $\bot$ (only no-answers):

contrariety $\mathrm{CT}(a,b)$	iff	$\mathbf{A}(a) \cap \mathbf{A}(b) = \bot$ and $\mathbf{A}(a) \cup \mathbf{A}(b) \neq \top$
contradiction $\mathrm{CD}(a,b)$	iff	$\mathbf{A}(a) \cap \mathbf{A}(b) = \bot$ and $\mathbf{A}(a) \cup \mathbf{A}(b) = \top$
subcontrariety $\mathrm{SCT}(a,b)$	iff	$\mathbf{A}(a) \cap \mathbf{A}(b) \neq \bot$ and $\mathbf{A}(a) \cup \mathbf{A}(b) = \top$
subalternation $\mathrm{SB}(a,b)$	iff	$\mathbf{A}(a) \cap \mathbf{A}(b) = \mathbf{A}(a)$ and $\mathbf{A}(a) \cup \mathbf{A}(b) = \mathbf{A}(b)$

Two notes are in order, in connection with the above definitions of opposition.
On the one hand, a minimal *number* of questions is required to preserve the relations of contrariety and subcontrariety between any objects a, b.

Proof. Let $i < 3$, e.g. $i = 2$ or $i = 1$. Suppose that $\mathbf{A}(a) = 10$. If $\mathbf{A}(b) = 00$, then $\mathrm{Op}(b,a) = \mathrm{SB}(b,a)$; if $\mathbf{A}(b) = 01$, then $\mathrm{Op}(a,b) = \mathrm{CD}(a,b)$; if $\mathbf{A}(b) = 11$, then $\mathrm{Op}(a,b) = \mathrm{SB}(a,b)$. No other relation occurs whenever $i = 2$, and, *a fortiori*, with $i < 2$. ■

The case where $i = 1$ corresponds to the usual truth-functional semantics where each proposition is given a unique value 1 (for True) and 0 (for False), and this is the reason why McCall rightly claimed in [10] that no other operator than a contradiction-forming one can be devised in it.
On the other hand, the above definitions betray a real difference between *subalternation* (in symbols: SB) and the other relations: not only does the former not hold when a and b are interchanged, since SB is not a *symmetrical* relation; but also, the $n = 4$ questions used to characterize opposition are not sufficient to identify SB. Indeed, the latter holds once every given question about a and b can be answered positively or negatively together; but an additional

condition must be added to it, to the effect that a given question cannot be answered negatively about b once answered positively about a. By omitting this further constraint, the result is a mere relation of non-contradiction or *independence* (see [2]) with respect to which subalternation is a subcase.

Proposition 11
Op(a,b) is a relation of *independence* IND iff:
IND(a,b) iff $\mathbf{A}(a) \cap \mathbf{A}(b) \neq \bot$ and $\mathbf{A}(a) \cup \mathbf{A}(b) \neq \top$

It may be replied that SB is not a relation of opposition at all, in the light of the preceding difficulty. For example, Demey & Smessaert argued in [5] that the Aristotelian square is a complex gathering of two different sorts of relation from two separate question-answer games, namely: opposition Op(a,b), and implication Imp(a,b). While Imp can be equated with the Tarskian relation of consequence Cn, we argue that subalternation can be embedded into the unique question-answer game defining logical oppositions (see section 4.2). By doing so, consequence is made a particular case of opposition in the sense that its very definition calls for the relation Op. More precisely, subalternation is formed by a kind of *double negation*. In accordance with our structuralist view of meaning as a synchronic set of different objects, let us see how negation takes in our algebraic logic of opposition.

3.3.3. Opposites as negations

As an alternative to the systematic treatment through sequent calculi (e.g. in [11]), Piaget paved the way to a general theory of negation by proposing in [12] a so-called *theory of reversibility* and its corresponding INRC Group of group-theoretical operations. In order to account for his genetic epistemology, Piaget claimed that intelligent reasoning consists in transforming structured elements with the help of a number of basic operations such as switch and permutation. A brief look at the former definitions of oppositions (see Definition 10) shows how reversibility is on a par with our main concern.

To begin with, Piaget's INRC Group is a set of 4 operations N,R,C, together with a trivial one I. Albeit restricted to the special case of binary propositions of classical logic, this whole device can be rendered within **QAS** as follows.

Proposition 12
Let $\mathbf{A}(a) = \langle \mathbf{a}_1(a),...,\mathbf{a}_n(a)\rangle$ be an arbitrary object individuated by n questions, and let § be a switching operation of denial that applies to single values $\mathbf{a}_i(a)$ such that §(1) = 0. Then the INRC Group can be defined by operations of switching and permuting upon every single value of $\mathbf{A}(a)$:

Identity I	(not switching, not permuting)	$\mathrm{I}(a) = \langle(\mathbf{a}_1(a),...,\mathbf{a}_n(a)\rangle$
Inversion N	(switching, not permuting)	$\mathrm{N}(a) = \langle §(\mathbf{a}_1(a)),...,§(\mathbf{a}_n(a))\rangle$
Reciprocity	(not switching, permuting)	$\mathrm{R}(a) = \langle(\mathbf{a}_n(a)),...,(\mathbf{a}_1(a))\rangle$
Correlation	(switching, permuting)	$\mathrm{C}(a) = \langle §(\mathbf{a}_n(a)),...,§(\mathbf{a}_1(a))\rangle$

Each of these operations can be obtained through a combination of other ones. Thus

Proposition 13
INRC Group includes the following rules of iteration:

Identity	II = NN = RR = CC = I
Commutation	For every X,Y $\in$ {I,N,R,C}, XY = YX
Idempotence	For every X $\in$ {I,N,R,C}, IX = X

Complementarity NR = C, NC = R, RC = N

While stressing the link between reversibility and the opposite-forming operators O, let us note the difference between the operations of *denial* and *negation*: the former is applied to single values, whereas the latter applies to whole structured values. Denial is a sort of *proto-negation* that helps to form logical negations, just as Humberstone suggested in [7] by proposing to iterate negation such that $\S\S a = {\sim}a$.

Moreover, N exactly matches with a contradiction-forming operator in that it proceeds by reverting any single value and thereby satisfies the definition of contradiction (see Definition 10). Nevertheless, there is no such one-one correspondence between each of the four operations of Piaget's INRC Group and the four opposite-forming operators op = {ct,cd,sct,sb}. Apart from the special case N(a) = cd(a), which opposite is constructed by R and C depends upon which logical value these reversibility operators are applied to. Taking **A**(a) = 1000 as an example, R(a) = 0001 = ct(a) and C(a) = 1110 = sb(a); while taking **A**(a) = 1100 entails that R(a) = 0011 = cd(a) and C(a) = 1100 = I(a).

More interestingly, negation can be characterized in two ways through our opposite-forming operators and, thus, in terms of opposition. First, more than three non-trivial operators like Piaget's ones can be devised to create opposite terms O(a) from a; it consists in applying the operator of denial to some single values of a but not all of them, the result of which is a distinction between *global* and *local* negations (see [15,16]). Second, such usual non-classical negations as *paracomplete* (intuitionist) and *paraconsistent* negations can be rendered within our logical theory of opposition. Starting from a result by Béziau [3], it has been shown that a logical hexagon of modal oppositions includes three sorts of logical negations, namely: classical negation is the contradiction-forming operator, whereas paracomplete and paraconsistent negations correspond to the contrary- and subcontrary-forming operators, respectively. More generally, a distinction is thus made between extensional and intensional negations.

Proposition 14
For any object a:
The contradiction-forming operator cd is an *extensional* operator of negation such that there is only one b resulting from cd(a) = b.
The contrary- and subcontrary-forming operators ct and sct are *intensional* operators of negation such that there are more than one b resulting from ct(a) = b.

Proof. By Proposition 10.
A logical negation is *paracomplete* iff the Law of Excluded Middle (LEM) fails with it, i.e. there is a logical negation O such that LEM: $a \vee$ O(a) is not tautological. Let **A**(a) ∪ **A**(O(a)) ≠ ⊤ the algebraic counterpart of the statement that LEM is not tautological in **QAS**. If **A**(a) ∪ **A**(O(a)) ≠ ⊤, then $\mathbf{a}_i(a) = \mathbf{a}_i(\mathrm{O}(a)) = 0$ for some single value $\mathbf{a}_i(a)$ of **A**(a). By definition of CT, **A**(a) ∪ **A**(b) ≠ ⊤ when ct(a) = b. Hence LEM fails if O = ct.
A logical negation is *paraconsistent* iff the Law of Explosion (LE) fails, i.e. there is a logical negation O such that, for any b, $a \wedge$ O(a) does not entail b. Let SB($a \wedge$ O(a),b) the counterpart of LE. The failure of LE is to be proved by a counterexample such that **A**($a \wedge$ O(a)) ∩ **A**(b) ≠ **A**($a \wedge$ O(a)), i.e. **A**($a \wedge$ O(a)) ≠ ⊥. By definition of SCT, **A**(a) ∩ **A**(b) ≠ ⊤when sct(a) = b. Hence LE fails if O = sct.

■

To sum up, the debate launched by Slater about the meaning of logical negation in [18] led to the construction of opposite-forming operators, doing justice to the occurrence of non-classical negations within the theory of opposition. Such a rationale had been foreshadowed by Piaget's

INRC Group, while noting again that the latter are to be clearly distinguished from the class op of opposite-forming operators (i.e. there is no one-one correspondence between the pairs {R,C} and {ct,sb}, respectively).

4. Objections (and its replies)

A number of objections can be raised against our whole enterprise, from the structuralist-minded view of meaning to the translation of standard logics into **QAS**. Let us see a sample of these, while attempting to give sufficient replies.

4.1. Opposition is nothing but incompatibility

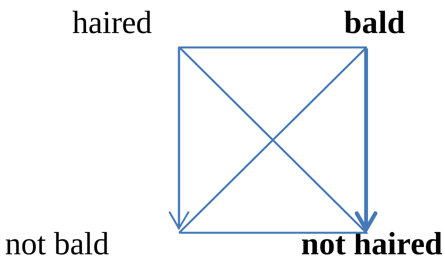
Aristotle claimed himself that subcontrariety is an opposition “only verbally”, in contrast to the genuine instances of contrariety and contradiction. This suggests that an Aristotelian opposition between any two sentences *a* and *b* is synonymous with *incompatibility*, in the sense that both cannot be true at once. If so, then our logical theory of opposition should be renamed as a theory of non-identity or, better, a theory of *difference* that accounts for the logical connections between different objects within a structured set of objects (possible worlds, or lexical fields).

A look at the Platonic process of “diaeresis” should argue for our case, however. Indeed, the dialectic process of definition can be seen as a diachronic question-answer game where different objects are more and more individuated by increasing the number of questions characterizing them. Moreover, it has been seen that the operator of denial § applies to a single Boolean value by switching it from 1 to 0 (and conversely), just as the contradiction-forming operator cd applies to ordered values.

In a nutshell, our algebraic view of logical values as structured bitstrings helps to explain why opposition produces the meaning of different objects without implying their mutual incompatibility. This also means that contradiction is the primary opposition underlying any other one, including the “verbal” case of subcontrariety and even subalternation.

4.2. Consequence is not subalternation

That a man is bald *entails* that it is not haired, in accordance to the contrary relation between “bald” and “haired”. Indeed, “not haired” is the contradictory of “haired” and, given that any contradictory of a contrary is a subaltern, the contradictory of the contrary of “bald”, “not haired”, stands for its subaltern.



In semi-formal words: ct(haired) = bald, and cd(bald) = not bald; hence cd(ct(haired)) = sb(haired) = not bald. This calculus is another evidence for the fact that Piaget’s reversibility operators differ from our opposite-forming operators, by passing, insofar as the latter are not commutative.

Proposition 15
Let ~ the symbol for classical negation, ¬ for paracomplete negation, and – for paraconsistent negation. Then:

15.1 $\sim(a) := \mathrm{cd}(a)$; $\neg(a) := \mathrm{ct}(a)$; $-(a) := \mathrm{sct}(a)$
15.2 subalternation results from the double mixed negation $\sim\neg(\mathrm{a}) := \mathrm{cd}(\mathrm{ct}(\mathrm{a}))$
15.3 the members of op are not *commutative* operators: for any $x,y \in \{\mathrm{ct},\mathrm{cd},\mathrm{sct},\mathrm{sb}\}$, $x(y(a)) \neq y(x(a))$ (where $x \neq y$)

A proof of 15.3 can be given thanks to the intensional behavior of the so-called non-classical negations, where there is a one-many mapping from the input value to the resulting opposite outputs (between brackets in the sequel).

Proof. By induction upon the members of the class op of opposing-forming operators.
Let $\mathbf{A}(a) = 1000$. Then:
$\mathrm{ct}(a) = \{0000,0100,0010,0001,0110,0011,0101\}$
$\mathrm{cd}(\mathrm{ct}(a)) = \{1111,1011,1101,1110,1001,1100,1010\}$
$\mathrm{cd}(a) = 0111$
$\mathrm{ct}(\mathrm{cd}(a)) = \varnothing$
Therefore $\mathrm{cd}(\mathrm{ct}(a)) \neq \mathrm{ct}(\mathrm{cd}(a))$.
(The reader is pleased to go through the entire inductive proof.) ■

The sole exception is the case where the iterated operator is the extensional case of contradiction, reproducing the classical law of double negation in **QAS**: $\mathrm{cd}(\mathrm{cd}(a)) := \sim\sim(a) = a$. It is obviously not so with non-classical negations, especially with the paracomplete operator that famously violates the aforementioned inference rule: $\mathrm{ct}(\mathrm{ct}(a)) \neq a$.

It could be replied to all of this that subalternation is nothing but a very restrictive counterpart of logical consequence. Whatever the case may be about the crucial properties of consequence, it is taken to be granted that our Boolean treatment is on a par with the semantic view of logical consequence as *truth-preservation*. Besides, the former helps to abstract from the notion of truth by claiming that any yes-answer to premises must lead to the same answers in the conclusion. In other words, any object occurs as a consequence whenever it confirms anything accepted about its premises. For this very reason, consequence, entailment, and subalternation are equated with each other from our point of view. Although there might be alternative views of consequence, let us argue that our **QAS** should be able to account for such non-standard versions by changing the central clauses of its question-answer game.

4.3. There is no calculus for opposite-forming functions

It has been noted in the preceding section that most of the opposite-forming operators proceed as one-many mappings, that is, operators with one input value and several output values. Mathematically speaking, this is a sufficient reason to establish that op is not a proper *function*: only one-one or many-one mappings are entitled to be called by this name, whereas one-many mappings do not. This is not a sufficient reason to conclude that no calculus can be devised for a theory of opposition and its constructive operators, however. Following the calculus of iterated negations by Kaneiwa [9], and by analogy with the arithmetic operation of square root, it clearly appears that $\sqrt{4}$ has a definite number of output values, i.e. $\sqrt{4} = \{-2,2\}$. In the same line, a definite number of values $b_1,\ldots,b_n$ can be assigned to any opposite of a such that $\mathrm{op}(a) = \{b_1,\ldots,b_n\}$. This calculus leads to a set op of *multifunctions* (or many-valued functions), instead of usual functions.

Admittedly, the resulting calculus is complicated by a more complex range of possible values. For instance, how many contraries of an increasing width of bitstrings there can be should be an increasing set of outputs … or the null set, in case the input value couldn't be said to have contraries at all. To clarify this complex situation, let us return to the structured values and their set-theoretical properties.

Proposition 16
Let Card be the symbol of cardinality. Then for any value of a, Card(cd(a)) = 1.

Proof. By Proposition 14, every logical object cannot have but one contradictory. Hence the cardinality of cd(a) is 1.

Proposition 17
Let m, n and y(a) be the number of answers, questions, and yes-answers in the logical value of a, respectively. Then for any a, Card(ct(a)) = $m^{n\text{-y}(a)}$ - 1.

Proof. By truncating the yes-answers **A**(a).
According to the definition of contrariety in Proposition 10, any yes-answer to a i^{th} question about a entails a corresponding no-answer for its contrary b. That is, $\mathbf{a}_i(b)$ = §(1) = 0 whenever $\mathbf{a}_i(a)$ = 1. By truncating every valuation where $\mathbf{a}_i(a)$ = 1, there remains a subset of n-y(a) cases with only no-answers for a, i.e. $\mathbf{a}_{n/i}(a)$ = 0. Then $\mathbf{a}_{n/i}(b)$ = 1 or 0, which yields a maximal number of possible valuations while excluding the special case with only $\mathbf{a}_{n/i}(a)$ = 1 (a and b would be contradictories, otherwise). As there are m^n possible valuations for m sorts of answers and n questions, the non-truncated bitstring of n-y(a) elements results in a set of $m^{n\text{-y}(a)}$ possible valuations minus the aforementioned excluded case with only yes-answers. Hence Card(ct(a)) = $m^{n\text{-y}(a)}$ - 1. ■

Example: let **A**(a) = 0100, with m = 2, n = 4, and y(a) = 1. Hence:
$\mathbf{a}_2(a)$ = 1, therefore $\mathbf{a}_2$(ct(a)) = §(1) = 0; by truncating the latter case, there remains a set of n-y(a) = 3 cases where $\mathbf{a}_{n/2}(a)$ = 1 or 0. That is:

	$\mathbf{a}_1(a)$	$\mathbf{a}_2(a)$	$\mathbf{a}_3(a)$	$\mathbf{a}_4(a)$	
A(a)	0	1	0	0	
A(ct(a))	0	0	0	0	(1)
	1	0	0	0	(2)
	0	0	1	0	(3)
	0	0	0	1	(4)
	1	0	1	0	(5)
	1	0	0	1	(6)
	0	0	1	1	(7)
	~~1~~	~~0~~	~~1~~	~~1~~	(= cd(a))

Card(ct(a)) = $m^{n\text{-y}(a)}$ - 1 = 2^{4-1} - 1 = 2^3 - 1 = 8 - 1 = 7, namely:
ct(a) = {0000,1000,0010,0001,1010,1001,0011}
Note: **A**(a) = 0100 and **A**(b) = 0000 stand into a relation of contrariety and subalternation at once, since we have both CT(a,b) = CT(b,a) and SB(b,a). This is allowed by the definitions of CT and SB, however (see Proposition 10), merely excluding the case where a and b cannot be *false* at once (by CT).

Proposition 18
Let m, n and y(a) be the number of answers, questions, and yes-answers in the logical value of a, respectively. Then for any a, Card(sct(a)) = $m^{\text{y}(a)}$ - 1.

Proof. By truncating the no-answers in **A**(a).

According to the definition of subcontrariety in Proposition 10, any no-answer to a i^{th} question about a entails a corresponding yes-answer for its subcontrary b. That is, $\mathbf{a}_i(b) = §(0) = 1$ whenever $\mathbf{a}_i(a) = 0$. By truncating every valuation where $\mathbf{a}_i(a) = 0$, it remains a subset of y(a) cases with only yes-answers for a, i.e. $\mathbf{a}_{n/i}(a) = 1$. Then $\mathbf{a}_{n/i}(b) = 1$ or 0, which yields a maximal number of possible valuations while excluding the special case with only $\mathbf{a}_{n/i}(a) = 0$ (a and b would be contradictories, otherwise). As there are m^n possible valuations for m sorts of answers and n questions, the non-truncated bitstring of y(a) elements results in a set of $m^{y(a)}$ possible valuations minus the aforementioned excluded case. Hence Card(sct(a)) = $m^{y(a)}$ - 1. ■

Example: let $\mathbf{A}(a)$ = 1011, with $m = 2$, $n = 4$, and y(a) = 3. Hence:
$\mathbf{a}_2(a) = 0$, therefore $\mathbf{a}_2$(sct(a)) = §(0) = 1; by truncating the latter case, there remains a set of n-y(a) = 3 cases where $\mathbf{a}_{n/2}(a) = 1$ or 0. That is:

	$\mathbf{a}_1(a)$	$\mathbf{a}_2(a)$	$\mathbf{a}_3(a)$	$\mathbf{a}_4(a)$	
$\mathbf{A}(a)$	1	0	1	1	
$\mathbf{A}$(sct(a))	1	1	1	1	(1)
	0	1	1	1	(2)
	1	1	0	1	(3)
	1	1	1	0	(4)
	0	1	0	1	(5)
	0	1	1	0	(6)
	1	1	0	0	(7)
	~~0~~	~~0~~	~~1~~	~~0~~	(= cd(a))

Card(sct(a)) = $m^{y(a)}$ - 1 = 2^3 - 1 = 8 - 1 = 7, namely:
sct(a) = {1111,0111,1101,1110,0101,0110,1100}

Note: $\mathbf{A}(a)$ = 1101 and $\mathbf{A}(b)$ = 1111 stand into a relation of subcontrariety and subalternation at once, since we have both SCT(a,b) = SCT(b,a) and SB(a,b). This is allowed by the definitions of SCT and SB (see Proposition 10), merely excluding the case where a and b cannot be *true* at once (by definition of SCT).

The above computations nicely match with the definition Aristotle gave to subcontraries as "contradictories of contraries" (see e.g. [3]). This plural expression should be clearly distinguished from the singular characterization of a subaltern as the "contradictory of a contrary".

Proposition 19
For any objects a,b:
19.1 a and b are *subcontrary* to each other iff their contradictories are contrary to each other, so that:
SCT(a,b) = CT(cd(a),cd(b))

Proof. According to Proposition 10, contradiction proceeds by switching every answer $\mathbf{a}_i(a)$ such that $\mathbf{a}_i$(cd(a)) = §($\mathbf{a}_i(a)$). According to Proposition 17 and Proposition 18, the non-truncated subsets of contraries and subcontraries are respectively such that $\mathbf{a}_{n/i}(a) = 0$ and $\mathbf{a}_{n/i}(a) = 1$, i.e. $\mathbf{a}_{n/i}(a)$ = §($\mathbf{a}_{n/i}(a)$). Now these are contradictory to each other. Therefore, SCT(a,b) = CT(cd(a),cd(b)). ■

19.2 b is a *subaltern* of a iff b is the contradictory of a contrary of a, so that:
Card(sb(a)) = Card(ct(a))

Proof. By Proposition 15.2, sb(a) = cd(ct(a)). There is only one contradictory of any opposite term op(a) of a, by Proposition 16: Card(cd(op(a))) = Card(op(a)), hence Card(sb(a)) = Card(ct(a)).

An alternative proof of the later result can be obtained through the definition of subalternation by Proposition 10: each yes-answer being preserved in the subaltern sb(a), truncate every yes-answer of a while excluding the case where $\mathbf{a}_{n/i}$(sb(a)) = 1 (a and sb(a) would be identical, otherwise). Thus compute the non-truncated bitstring of no-answers as $m^{n\text{-y}(a)} - 1$.

Example: let $\mathbf{A}(a)$ = 0100, with $m = 2$, $n = 4$, and y(a) = 3. Hence:
$\mathbf{a}_2(a) = 1$, therefore $\mathbf{a}_2(\text{sb}(a)) = 1$; by truncating the latter case, there remains a set of n-y(a) = 3 cases where $\mathbf{a}_{n/2}(a)$ = 1 or 0. That is:

	$\mathbf{a}_1(a)$	$\mathbf{a}_2(a)$	$\mathbf{a}_3(a)$	$\mathbf{a}_4(a)$	
$\mathbf{A}(a)$	0	1	0	0	
$\mathbf{A}(\text{sb}(a))$	1	1	0	0	(1)
	0	1	1	0	(2)
	0	1	0	1	(3)
	1	1	1	0	(4)
	1	1	0	1	(5)
	0	1	1	1	(6)
	1	1	1	1	(7)
	~~0~~	~~1~~	~~0~~	~~0~~	(= a)

Card(sb(a)) = $m^{\text{y}(a)} - 1 = 2^3 - 1 = 8 - 1 = 7$, namely:
sct(a) = {1100,0110,0101,1110,1101,0111,1111}

5. Conclusion

The gist of the present paper relied upon an algebraic analysis of opposition, in the name of a structural view of meaning. Not everything has been said about it, admittedly: although logical consequence is depicted as a by-product of the larger relation of opposition, no counterpart of Tarski's systematic work about consequence is available until now with respect to opposition.

This should lead to a twofold investigation in later works. Firstly, a general theory of iterated oppositions for n iterations, to generalize the above section 4.3 and its multifunctional calculus of opposites: what can be the contrary of the subcontrary of the subaltern of some object a, for instance? Secondly, the construction of an abstract operator of opposition in line with Tarski's operator of consequence (see especially [21]): can there be such an operator to be characterized either in logic, or algebra, or topology?

Whether what has been displayed in the paper belongs to the area of algebra or logic of opposition is questionable. For one thing, our formal theory of opposition crucially relies upon Boolean bitstrings, and this has much more to do with algebra than logic. At the same time, such a distinction between logic and algebra assumes that the former be considered as a pair $\langle$L,Cn$\rangle$ including a formal language (set of formulas) L and a basic operator of consequence Cn upon elements of L. A next step towards a more comprehensive approach of logic would consist in embedding logical consequence within a broader pair $\langle$L,Op$\rangle$, accordingly: just as consequence has been investigated in the form of either a relation or an operator [20,21], opposition could be viewed from the perspective of a general relation Op or a general opposite-forming operator op.

Finally, our treatment of meaning through Boolean translations of information amounts to a finitist version of possible-world semantics, i.e. an algebraic semantics where models are finite sets of sets of objects. Meaning as a set of lexical fields is thus treated by a finite set of overlapping question-answer games about definite objects. If so, then whoever aspiring to a general model theory should blame **QAS** for limiting the use of logic to finitely many models. Two replies could be given in turn: if finite question-answer games lead to finitely many-valued sets of objects, then their infinite counterparts might lead to infinitely many-valued objects (by analogy to the infinitely many-valued matrices); eventually, our constructive treatment of meaning as a questioning process is played by bona fide speakers who don't practice with infinite set of data. For who plays with infinity, if not God (if any)?

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Modeling Multistage Decision Processes with Reflexive Game Theory

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Abstract:
Reflexive Game Theory (RGT) has been designed to model a single session of decision making in a group. On the other hand, often one final decision is a result of series of discussions, which are dedicated to various aspects of the final decision. This paper introduces approach to model such multi-stage decision making processes by means of RGT. The basic idea is to set particular parameters (group structure, mutual relationships and influencess, etc.) during the series of decision making sessions. To illustrate this idea three examples are provided.

1. Introduction

The Reflexive Game Theory (RGT) [1, 2] allows to predict choices of subjects in the group. To do so, the information about a group structure and mutual influences between subjects is needed. Formulation and development of RGT was possible due to fundamental psychological research in the field of *reflexion*, which had been conducted by Vladimir Lefebvre [3].

The group structure means the set of pair-wise relationships between subjects in the group. These relationships can be either of alliance or conflict type. The mutual influences are formulated in terms of elements of Boolean algebra, which is build upon the set of universal actions. The elements of Boolean algebra represent all possible choices. The mutual influences are presented in the form of Influence matrix.

In general, RGT inference can be presented as a sequence of the following steps [1]:

1) formalize choices in terms of elements of Boolean algebra of alternatives;
2) present of a group in the form of a fully connected *relationship graph*, where solid-line and dashed-line ribs (edges) represent alliance and conflict relationships, respectively;
3) if relationship graph is decomposable, then it should be represented in the form of polynomial: alliance and conflict are denoted by conjunction (·) and disjunction (+) operations;
4) perform diagonal form transformation (build diagonal form on the basis of the polynomial and fold this diagonal form);
5) deduct the decision equations;
6) input influence values into the decision equations for each subject.

Let us call the process of decision making in a group to be a *session*. Therefore, RGT inference is a single session.

2. Model of Two-Stage Decision Making: Formation of Points of View

This study is dedicated to the matter of setting mutual influences in a group by means of *reflexive control.* [4]

The influences, which subjects make on each other, could be considered as a result of a decision making session previous to *ultimate decision making* (*final session*). The influences of this type we would call *set-up influences*. The set-up influences are intermediate result of the overall decision making process. The term set-up influences is related to the influences, which are used during the final session, only.

Consequently, the overall decision making process could be segregated into two stages. Let the result of such discussion (decision making) be a particular decision regarding the matter under consideration. We assume that actual decision making regarding the matter of interest (final session – Stage 2) is preceded by *preliminary session* (Stage 1), which is about a decision making regarding the influences (points of view), which each subject will support during the final session. Such overall decision making process we call *two-stage decision making process*. The general schema of the two-stage decision making is presented in Fig.1.

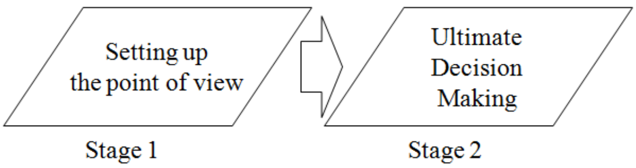

Fig. 1. The general schema of the two-stage decision making.

To illustrate such model we consider a simple example.

Example 1. Let director of some company has a meeting with his advisors. The goal of this meeting is to make decision about marketing policy for the next half a year. The background analysis and predictions of experts suggest three distinct strategies: aggressive (action α), moderate (action β) and soft (action γ) strategies.

The points of view of director and his advisors are formulated in terms of Boolean algebra of alternatives. Term *point of view* implies that a subject makes the same influences on the others. Director supports moderate strategy ($\{\beta\}$), the first and the second advisors are supporting aggressive strategy ($\{\alpha\}$), and the third advisor defends the idea of soft strategy ($\{\gamma\}$). The matrix of initial influences is presented in Table 1.

	a	b	b	d
a	a	$\{\alpha\}$	$\{\alpha\}$	$\{\alpha\}$
b	$\{\alpha\}$	b	$\{\alpha\}$	$\{\alpha\}$
c	$\{\beta\}$	$\{\beta\}$	c	$\{\beta\}$
d	$\{\gamma\}$	$\{\gamma\}$	$\{\gamma\}$	d

Table 1. Matrix of initial points of view (influences) used in Example 1.

Let director is in a conflict with all his advisors, but his advisors are in alliance with each other. Variable c represents the Director, variables a, b and d correspond to the 1st , the 2nd and the 3rd advisor, respectively.

The relationship graph is presented in Fig.2. Polynomial $abd+c$ corresponds to this graph.

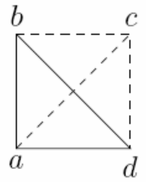

Fig. 2. Relationship graph for a director-advisors group.

After diagonal form transformation the polynomial does not change:

$$\begin{array}{lll} & & [a][b][d] \\ & [abd] & \quad + [c] \\ [abd+c] & & \quad = abd + c \end{array}$$

Then we obtain four decision equation and their solutions (decision intervals) (Table 2):

Subject	Decision Equations	Decision Intervals
a	$a = (bd+c)a + c\bar{a}$	$(bd+c) \supseteq a \supseteq c$
b	$b = (ad+c)b + c\bar{b}$	$(ad+c) \supseteq b \supseteq c$
c	$c = c + abd\bar{c}$	$1 \supseteq c \supseteq abd$
d	$d = (ab+c)d + c\bar{d}$	$(ab+c) \supseteq d \supseteq c$

Table 2. Decision equation and their solutions for Example 1.

Next we calculate the decision intervals by using information from the influence matrix:
subject a: $(bd+c) \supseteq a \supseteq c \Rightarrow (\{\alpha\}\{\gamma\}+\{\beta\}) \supseteq a \supseteq \{\beta\} \Rightarrow a = \{\beta\}$;
subject b: $(ad+c) \supseteq b \supseteq c \Rightarrow (\{\alpha\}\{\gamma\}+\{\beta\}) \supseteq b \supseteq \{\beta\} \Rightarrow b = \{\beta\}$;
subject c: $1 \supseteq c \supseteq abd \Rightarrow 1 \supseteq c \supseteq \{\alpha\}\{\alpha\}\{\gamma\} \Rightarrow 1 \supseteq c \supseteq 0 \Rightarrow c = c$;
subject d: $(ab+c) \supseteq d \supseteq c \Rightarrow (\{\alpha\}\{\alpha\}+\{\beta\}) \supseteq d \supseteq \{\beta\} \Rightarrow \{\alpha,\beta\} \supseteq d \supseteq \{\beta\}$.
Therefore, after the preliminary sessions, the points of view of the subjects have changed.

Director has obtained a freedom of choice, since he can choose any alternative: $1 \supseteq c \supseteq 0 \Rightarrow c = c$. At the same time, the 1st and the 2nd advisors support moderate strategy ($a = b = \{\beta\}$). Finally, the 3rd advisor now can choose between points of view $\{\alpha,\beta\}$ (aggressive of moderate strategy) and $\{\beta\}$ (moderate strategy) ($\{\alpha,\beta\} \supseteq d \supseteq \{\beta\}$).

Thus the points of view of the 1st and the second advisors are definite, while the point of view of 3rd advisor is probabilistic.

Next we calculate choice of each subject during the final session considering the influences resulting from the preliminary session. The matrix of set-up influences is presented in Table 3. The intervals in matrix imply that a subject can choose either of alternatives from the given interval as a point of view.

	a	*b*	*c*	*d*
a	*a*	$\{\beta\}$	$\{\beta\}$	$\{\beta\}$
b	$\{\beta\}$	*b*	$\{\beta\}$	$\{\beta\}$
c	$1 \supseteq c \supseteq 0$	$1 \supseteq c \supseteq 0$	*C*	$1 \supseteq c \supseteq 0$
d	$\{\alpha,\beta\} \supseteq d \supseteq \{\beta\}$	$\{\alpha,\beta\} \supseteq d \supseteq \{\beta\}$	$\{\alpha,\beta\} \supseteq d \supseteq \{\beta\}$	*d*

Table 3. The matrix of set-up influences for Example 1.

Subject *a*: $d = \{\alpha,\beta\}$: $(bd+c) \supseteq a \supseteq c \Rightarrow \{\beta\}\{\alpha,\beta\}+c \supseteq a \supseteq c \Rightarrow \{\beta\}+c \supseteq a \supseteq c$;
$d = \{\beta\}$: $(bd+c) \supseteq a \supseteq c \Rightarrow \{\beta\}\{\beta\}+c \supseteq a \supseteq c \Rightarrow \{\beta\}+c \supseteq a \supseteq c$.

Subject *b*: $d = \{\alpha,\beta\}$: $(ad+c) \supseteq b \supseteq c \Rightarrow (\{\beta\}\{\alpha,\beta\}+c) \supseteq b \supseteq c \Rightarrow \{\beta\}+c \supseteq b \supseteq c$;
$d = \{\beta\}$: $(bd+c) \supseteq a \supseteq c \Rightarrow \{\beta\}\{\beta\}+c \supseteq a \supseteq c \Rightarrow \{\beta\}+c \supseteq b \supseteq c$.

Subject *c*: $d = \{\alpha,\beta\}$: $1 \supseteq c \supseteq abd \Rightarrow 1 \supseteq c \supseteq \{\beta\}\{\beta\}\{\alpha,\beta\} \Rightarrow 1 \supseteq c \supseteq \{\beta\}$;
$d = \{\beta\}$: $1 \supseteq c \supseteq abd \Rightarrow 1 \supseteq c \supseteq \{\beta\}\{\beta\}\{\beta\} \Rightarrow 1 \supseteq c \supseteq \{\beta\}$.

Subject *d*: $(\{\beta\}\{\beta\}+c) \supseteq d \supseteq c \Rightarrow (\{\alpha\}\{\alpha\}+\{\beta\}) \supseteq d \supseteq \{\beta\} \Rightarrow \{\beta\}+c \supseteq d \supseteq c$.

Now we compare the results of a single session with the ones of the two-stage decision making.

The single session case has been considered above. Therefore if the final decision have been made after the single session, then the 3rd advisor would be able to choose alternative $\{\alpha,\beta\}$ and realize action α. This option implies that each advisor is responsible for a particular part of the entire company and can take management decisions on his own.

Next we consider the decision made after the two-stage decision making. In such a case, regardless of influence of the 3rd advisor (subject *d*), decisions of advisors *a* and *b* are defined by the interval $\{\beta\}+c \supseteq x \supseteq c$, where *x* is either *a* or *b* variable. Thus, if director is inactive ($c=0$), subjects *a* and *b* can choose either moderate strategy ($\{\beta\}$) or make no decision ($0=\{\}$). The same is true for subject *d*.

If the director makes influence $\{\beta\}$, then all the advisors will choose alternative $\{\beta\}$.

The director himself can choose from the interval $1 \supseteq c \supseteq \{\beta\}$ after the final session. This means that a director can choose any alternative containing action β. Thus, occasionally the director can realize his initial point of view as moderate strategy.

This example illustrates how using the two-stage decision making it is possible to make one's opponents choose the one's point of view. Meanwhile a person interested in such reflexive control can still sustain the initial point of view.

The obtained results are applicable in both cases when 1) only a director makes a decision; or 2) the decision are made individually by each subject.

3. A Model of a Multi-Stage Decision Making: Set-Up Parameters of the Final Session

Now we consider the two-stage model in more details. In the considered example, during the preliminary session only the decision regarding the influences has been under consideration. In general case, however, before the final session has begun, there can be made decisions regarding any parameters of the final session. Such parameters include but are not limit to:

1) group structure (relationships between subjects in a group);
2) points of view;
3) decision to start a final session (a time when the final session should start), etc.

We call the decisions regarding a single parameter to be *consecutive* decisions, and decisions regarding distinct parameters to be *parallel*.

Therefore, during the first stage (before the final session) it is possible to make multiple decisions regarding various parameters of the final session. These decisions could be both parallel and consecutive ones. Such model of decision making we call *multi-stage process of decision making* (Fig.3).

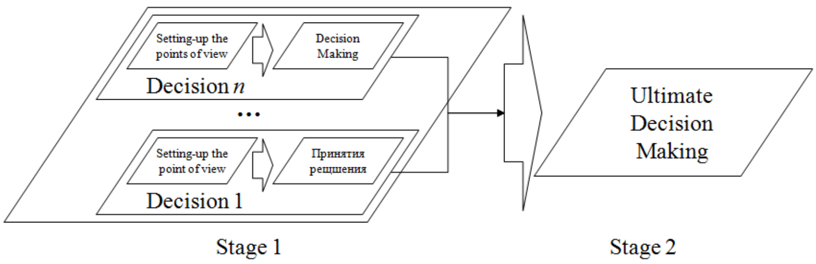

Fig. 3. Multi-stage decision making model.

4. Modeling Multi-Stage Decision Making Processes with RGT

Next we consider realization of multi-stage decision making with RGT.

Example 2: Change a group structure. Considering the subjects from Example 1, we analyze the case when director wants to exclude the 3rd advisor from the group which would make the final decision.

In such a case there is a single action – 1 – to exclude subject *d* from the group. Then Boolean algebra of alternatives includes only two elements: 1 и 0. Furthermore, it is enough that director just raise a question to exclude subject *d* from a group and make influence 1 on each subject: if $c = 1$, then a=1, b=1 and d=1 (Table 2). Thus the decision to exclude subject *d* from the group would be made automatically (Fig. 4).

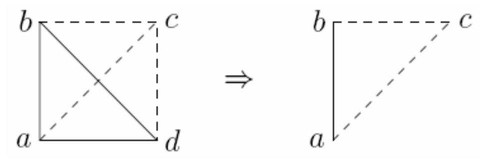

Fig. 4. Exclusion of a subject *d* from a group.

Example 3: Realization of a multi-stage decision making. Let the first decision discussed during the first stage is a decision regarding influences (points of view). The next decision was about exclusion of a subject *d* from the group. Thus, during the first step the formation of points of view has been implemented, then the structure of a group was changed. Therefore the group, which should make a final decision is described by polynomial $ab + c$. The decision equations and their solutions are presented in Table 4.

The overall multi-stage decision making process is presented in Fig.5.

Subject	Decision Equation	Decision Interval
a	$a = (b+c)a + c\bar{a}$	$(b+c) \supseteq a \supseteq c$
b	$b = (a+c)b + c\bar{b}$	$(a+c) \supseteq b \supseteq c$
c	$c = c + ab\bar{c}$	$1 \supseteq c \supseteq ab$

Table 4. Decision equations and decision interval for Example 3.

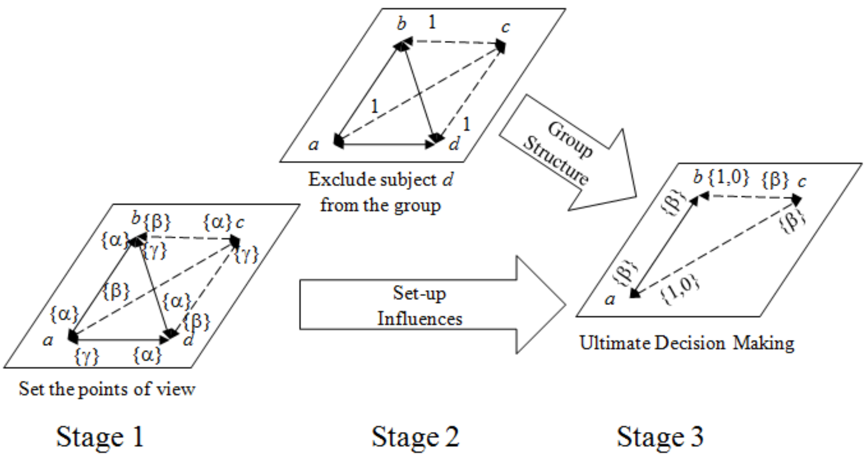

Fig. 5. Illustration of multi-stage decision making process. The influences are indicated by the arrow-ends of the ribs. The actual influence is presented near the arrow-end.

We consider that subject cannot change a point of view without preliminary session regarding the parameter. Therefore we assume that the points of view stay the same even after the group structure is changed.

Therefore, during the final session the subjects would make the set-up influences derived from the preliminary session: subjects a and b will make influences $\{\beta\}$ and $1 \supseteq c \supseteq \{\beta\}$, respectively.

Such process is introduced in Fig. 5. During the 1st stage (the first step), the points of view of subjects have been formed. On the 2nd stage (the second step), the decision to exclude subject d from a group has been made. Finally, during the 3rd stage the final decision regarding the marketing strategy has been made.

5. Discussion and Conclusion

This study introduces the two-stage and multi-stage decision making processes. During the first stage the decisions regarding the parameters of a final session are considered. The intermediate decisions are made during the preliminary sessions, while the final decision is made during the final session.

This study shows how before the final decision making the intermediate decision regarding parameters of the final session can be made and how the overall process of decision making could be represented as a sequence of decision making sessions.

This approach enables complex decisions which involve numerous parameters.

The important feature of the multi-stage decision making is that during the preliminary sessions subjects can convince other subjects to accept their own point of view. Therefore other subjects can be convinced to make decisions beneficial for a particular one. Such approach also allows to distribute the responsibility between all the members of the group, who make the final decision.

The result presented in this study allows to extend the scope of applications of RGT to modeling multi-stage decision making processes. Therefore it becomes possible to perform scenario analysis of various variants of future trends and apply reflexive control to the management of projects.

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The Light from the East

George Kiraz was born in Bethlehem to a Syriac Orthodox merchants family. He learned Syriac at the St. Mary's Church in Bethlehem and St. Mark's Monastery in Jerusalem. In 1983 he emigrated with his family to the United States. He obtained a master's degree in Syriac Studies from the University of Oxford under Dr. Sebastian Brock and a doctorate in Computational Linguistics from the University of Cambridge. He came back to the US in 1996 where he worked as a research scientist at Bell Laboratories, Lucent Technologies. He founded in 1992 Beth Mardutho: The Syriac Institute (formerly The Syriac Computing Institute). In 2001 he founded Gorgias Press, an academic publisher of books and journals covering a range of religious and language studies that include Syriac language, Eastern Christianity, Ancient Near East, Arabic and Islam, Early Christianity, Judaism, and more. He is the author of many works on Syriac studies including a six-volume *Concordance to the Syriac New Testament* (1993), a four-volume *Comparative Edition of the Syriac Gospels* (1996), etc.

Andrew Schumann: You are one of the best experts in Syriac studies. In many respects these studies are connected with Eastern Christianity. What could these studies give Christianity at all taking into account the fact that Greek is considered the original language of Christianity?

George Kiraz: Sebastian Brock of Oxford always talks about the three 'linguistic' pillars upon which Christianity is founded: The Latin West and the Greek East are the best known, but he then emphasizes the Syriac Orient, especially in its non-Hellenized form of Christianity. We forget in the West that Christianity began in the East and the Eastern fathers, both Byzantine and Semitic (in the form of Syriac) built the foundations of Christianity.

Andrew Schumann: Mar Eshai Shimun XXIII formulated the official position of the Assyrian Church of the East in 1957 that the Syriac Peshitta is the original of the New Testament. This view was popularized by the Assyrian scholar George Lamsa. Whether we can claim that the Aramaic Peshitta is the closest text to the original New Testament? As I know, the Church you belong to uses the Syriac Peshitta as the main source, as well.

George Kiraz: The church that I belong to, the Syriac Orthodox Church, also uses the Peshitta, so do all the Churches of the Syriac linguistic family including the Assyrian Church of the East, the Chaldean Church, the Syriac Catholic Church, the Maronite Church, and the Churches of the St. Thomas Christians in India. The Peshitta is one of the most ancient versions of the Bible. Its New Testament has many unique readings. The Peshitta New Testament is a revision of an older Syriac version called the "Old Syriac". There was another Gospel harmony, the Diattessaron, which is called in Syriac "the mixed Gospel". Scholars today agree that the Peshitta New Testament as we have it today is a translation from the Greek. But because it is very ancient, its readings are very important and differ from time to time from the Greek "received text". The Antioch Bible project brings a modern English translation of the Syriac Bible so that English speakers can appreciate the Syriac biblical text.

Andrew Schumann: His Eminence Mor Cyril Aphrem Karim ordained you Deacon (Ewangeloyo) in a grand ceremony at St. Mark's Cathedral which included numerous clergy and deacons from the archdiocese. His Eminence praised you for the contributions to the Syriac Orthodox worldwide community. For me it is a significant symbol that your scientific work in Syriac Studies is evaluated as a kind of spiritual mission of Ewangeloyo. How far do you feel that you have a special mission?

George Kiraz: It is difficult to separate the scholarly mission from the Church mission as Syriac is rooted in a Christian heritage. Having said that, one of course maintains all scholarly integrity when doing research. My mission is to serve the Syriac heritage in all its aspects, both secularly and religious. The ordination demonstrates the appreciation of the hierarchy to the scholarly work that is being done.

Andrew Schumann: You have studied Syriac from the point of view of religious studies as well as of computational linguistics, e.g., on the one hand, you published the book *Comparative Edition of the Syriac Gospels: Aligning the Old Syriac (Sinaiticus, Curetonianus), Peshitta and Harklean Versions* (1996), and, on the other hand, you designed the first computer fonts for Syriac in 1986 and later you designed Syriac fonts that became the most useful till now. What does this combination of religious and computational aspects give?

George Kiraz: It is the blend of computational linguistics and Syriac studies that excites me when I do my work. It is a weird combination, but I managed to make it work for me and for the projects I am involved in. The computational power allows me to do things faster and in unprecedented speed and efficiency. Having been raised in a Christian environment and Syriac being what it is, I see my work that stems from Syriac and computing united under one umbrella. Using the metaphor that miaphysites use for the description of the Nature of Christ: it is like an iron put on fire. Once you pick it up, it is difficult to separate the fire from the metal. I see computing and Syriac blending together.

Andrew Schumann: How do you understand Heidegger's famous phrase 'Die Sprache ist das Haus des Seins' in relation to Syriac? What is it the Syriac home, the Syriac universe? How does it differ from other universes?

George Kiraz: Language lies at the heart of one's identity. This cannot be emphasized more in the case of Syriac. Today, Syriac Christians are spread all over the world and Syriac Christianity is in danger. The more turbulent the Middle East becomes, the more we see immigrants leaving the Middle East to the West. But in the west, we know that no community can survive for more than 3 or 4 generations. It is with the use of the Syriac language as an identity tool that we can try to prolong the life of Syriac Christianity. I speak Classical Syriac with my kids, and they answer me in a blend of Syriac-English language. It is crazy but gives them the sense that they have a Syriac identity.

Andrew Schumann: Could you please tell us about some present and future projects of Beth Mardutho directed by you?

George Kiraz: Beth Mardutho publishes the peer-reviewed academic journal *Hugoye: Journal of Syriac Studies*. Recently, it finished publishing the first Syriac encyclopaedia title the *Gorgias*

Encyclopaedic Dictionary of the Syriac Heritage. We are now building the content of the Beth Mardutho Research Library. We have collected thousands of printed books, eBooks, images, etc. We would like to make all of this content available online, but we are in need of funding to do that.

Andrew Schumann: Gorgias Press, the academic publisher directed by you, is well known among scholars in Syriac language, Eastern Christianity, Ancient Near East, Arabic and Islam, Early Christianity, Judaism and so on. Maybe can you tell us about any future plans of this publisher?

George Kiraz: Our largest project now is the Antioch Bible which I have mentioned earlier. The Antioch Bible is a bilingual edition: Syriac and English. The English is an idiomatic translation of the Syriac with enough annotations to give variant translations. The Syriac text is fully vocalized and pointed. This is the first time in history where a fully vocalized Syriac text of the entire Bible is published in the Serto script with an English translation. We expect the project to be completed within seven years. We currently put out about 4-6 volumes every year. People can subscribe to the set on the Gorgias web site www.gorgiaspress.com/AntiochBible. We continue to publish books in all the fields that you have mentioned. We champion minority subject areas. That is our strength.